

---

# Precise Kinematic GPS Processing and Rigorous Modeling of GPS in a Block Adjustment

Martin Schmitz, Gerhard Wübbena

Geo++<sup>®</sup>

*Gesellschaft für satellitengestützte geodätische und  
navigatorische Technologien mbH*

D-30827 Garbsen, Germany

[www.geopp.com](http://www.geopp.com)

# Content

---



- Motivation
- Kinematic GPS Processing
  - GPS Error Sources
  - Systematic GPS Coordinate Errors
- Shift- and Drift Approximation in Combined GPS/Block Adjustment
- Rigorous GPS Model in Combined GPS/Block Adjustment
- Benefit of Rigorous Modeling of GPS
- Data Flow GEONAP/BINGO
- Summary

# Motivation

---

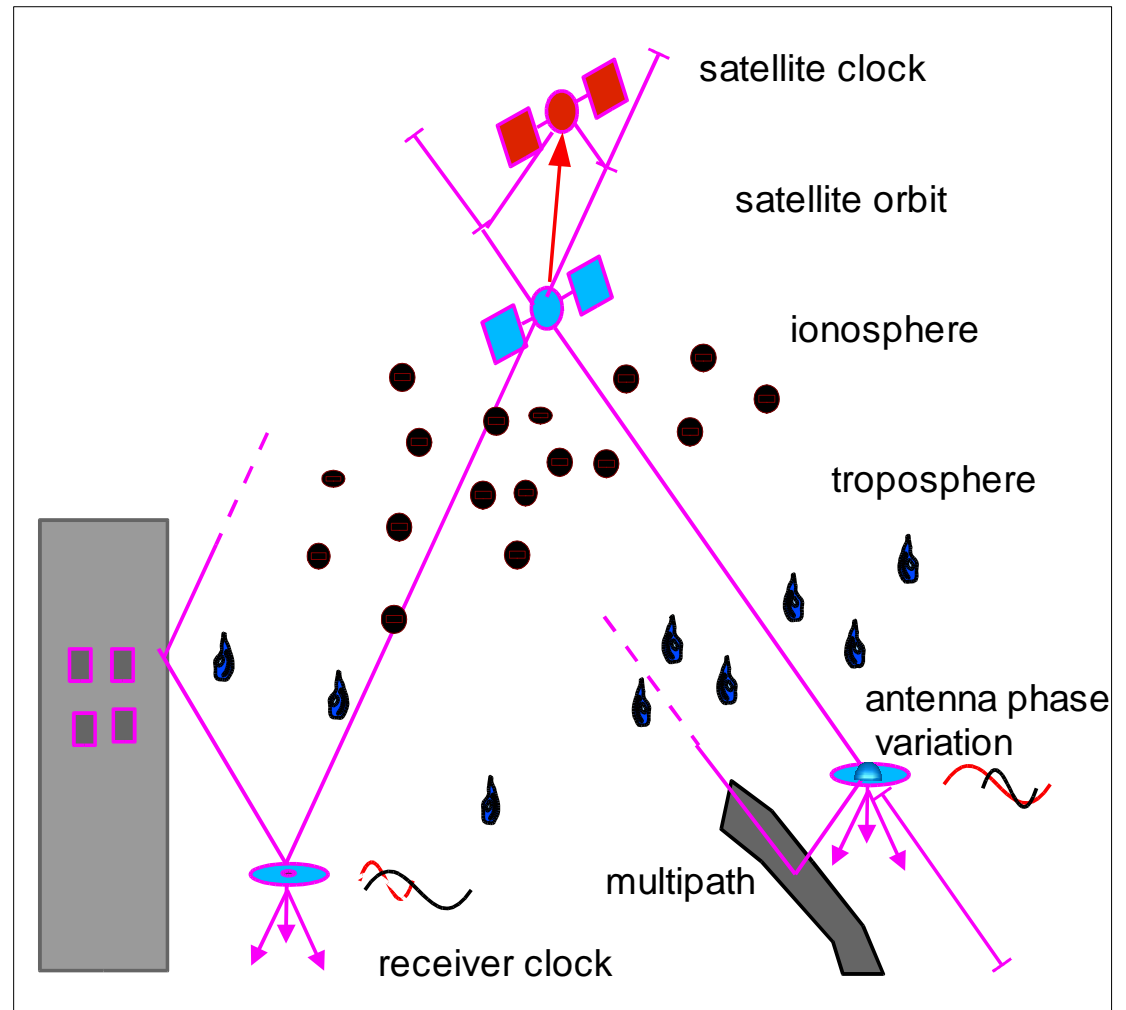


- precise kinematic GPS processing
- challenging GPS task
- combination of GPS and photogrammetric applications
- handling of remaining GPS coordinate errors
- generally approximation without reflecting GPS model applied
- rigorous GPS Model developed for combined GPS/block adjustment
- GEONAP-K GPS- / BINGO block adjustment
- operationally applied since 1996
- operational experiences underline advantages
  
- reconsideration of models required

# GPS Error Sources



- **station dependent error**
  - antenna phase variation (PCV)
  - multipath
- **distance dependent error**
  - ionosphere
  - troposphere
  - satellite orbit
- **kinematic GPS: additional systematic coordinate effects**
  - constellation changes
  - approximate or false ambiguity resolution



# Magnitude of GPS Error Sources



error source	absolute influence	relative influence
satellite orbit	2 ... 50 m	0.1 ... 2 ppm
clock	2 ... 100 m	0.0 ppm
ionosphere	0.5 ... >100 m	1 ... 50 ppm
troposphere	0.01 ... 0.5 m	0 ... 3 ppm
multipath code	m	m
multipath phase	mm ... cm	mm ... cm
antenna	mm ... cm	mm ... cm

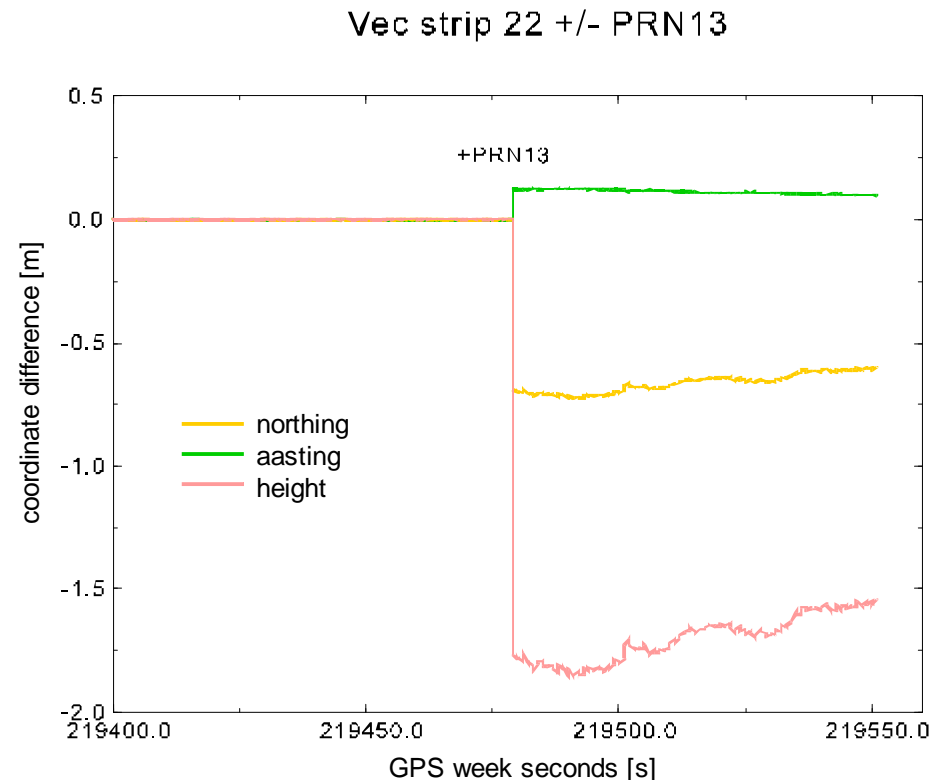
 high spatial correlation

 local (calibration)

# Systematic GPS Coordinate Errors



- systematic GPS coordinate errors may be present due to
  - false ambiguity fixing (time dependent effects)
  - changes in the satellite constellation (discontinuity)
  - GPS error sources
- magnitude of errors depends on
  - geometric GPS conditions (DOP values)
  - known systematic GPS effects
- modeling of these errors attempted in combined adjustment of GPS and aerial triangulation



# Modeling of Kinematic GPS



<i><b>Kinematic GPS Processing</b></i>	<i><b>local reference station</b></i>	<i><b>remote reference station</b></i>	<i><b>reference station network</b></i>
ambiguity resolution	possible	difficult	possible
distance dependent errors:			
• ionosphere	ignore, <b>estimate</b> , eliminate	ignore, <b>estimate</b> , eliminate	<b>estimate</b> , eliminate
• troposphere	<b>model</b> , estimate	<b>model</b> , estimate	<b>model</b> , estimate
• orbit	BE, <b>PE</b>	BE, model, <b>PE</b>	BE, model, <b>PE</b>
remaining systematic effects:			
• shift, drift of coordinates	approximate, <b>rigorous model</b>	approximate, <b>rigorous model</b>	approximate, <b>rigorous model</b>
• antenna PCV	<b>correct</b>	<b>correct</b>	<b>correct</b>
costs	high	low	low

- all major effects can be corrected or modeled
- consider costs for choice on reference station(s)

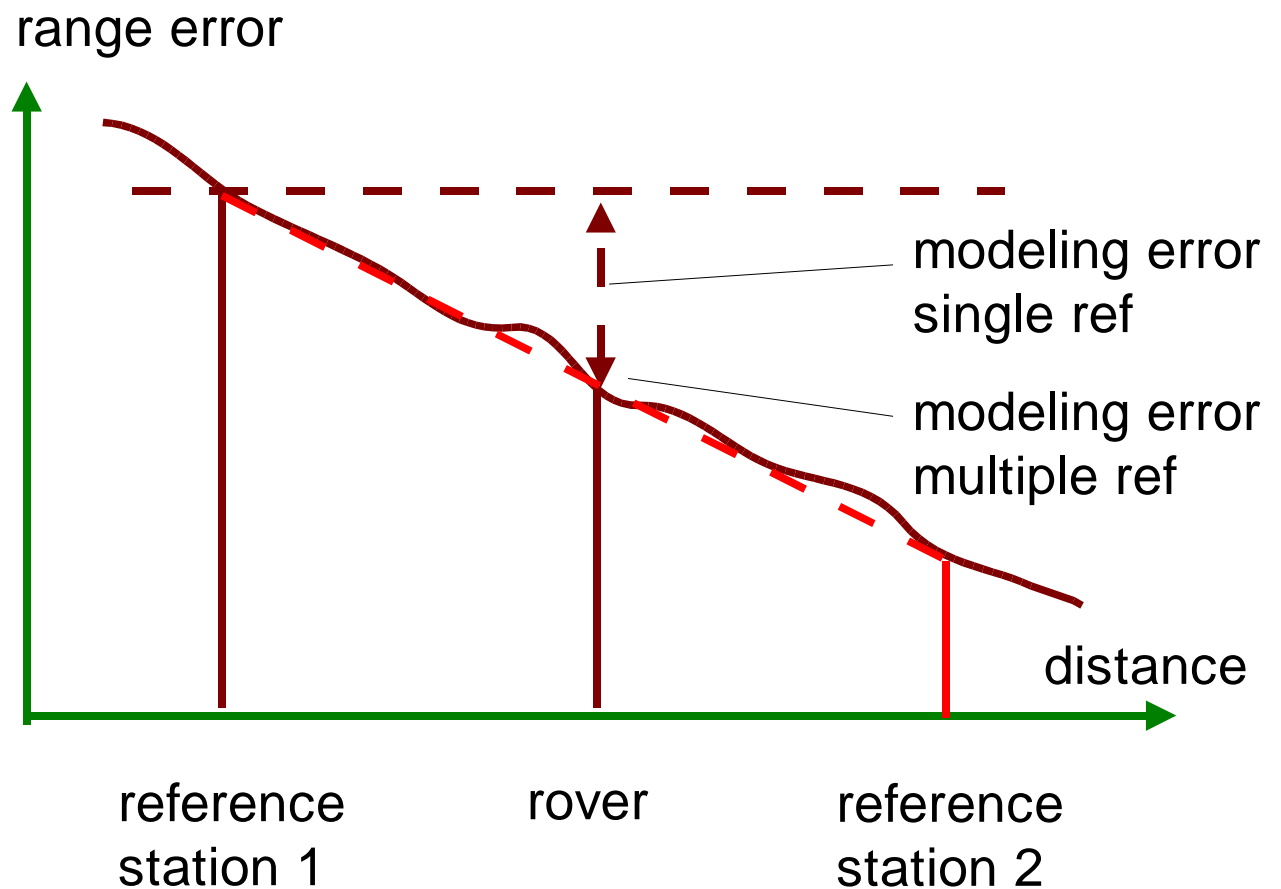
**■** recommended, but depends very much on application and data

BE broadcast ephemeris  
PE precise ephemeris  
PCV phase (center) variations

# Distance Dependent GPS Errors



- spatial correlation of GPS errors
- distance dependency
- advantage to use multiple reference stations
- allows better modeling of GPS errors





# Geo++<sup>®</sup> GEONAP

---



- GEONAP – since 1988
  - *Geodetic Navstar Positioning*
  - **multi-signal, multi-station**, multi-session adjustment (rigorous adjustment of different signals and multiple kinematic and/or static stations)
  - **undifferenced observable** with **complete variance-covariance estimation**
- development and maintenance of GEONAP by Geo++<sup>®</sup> - since 1990
- advanced GPS software
  - for static and kinematic applications
  - for small, large and regional applications
  - different accuracy levels from mm ... m



## DGPS

- trajectory with **dm ... m** accuracy
- precise navigation applications
  - **no ambiguity resolution** necessary
  - use of code observations
  - carrier-phase smoothed code observations

## PDGPS

- trajectory with **cm ... dm** accuracy
- precise positioning applications
  - **ambiguity resolution** required
  - simultaneous adjustment of kinematic and reference station data
  - several kinematic and/or reference stations possible

# Geo++<sup>®</sup> GEONAP

---

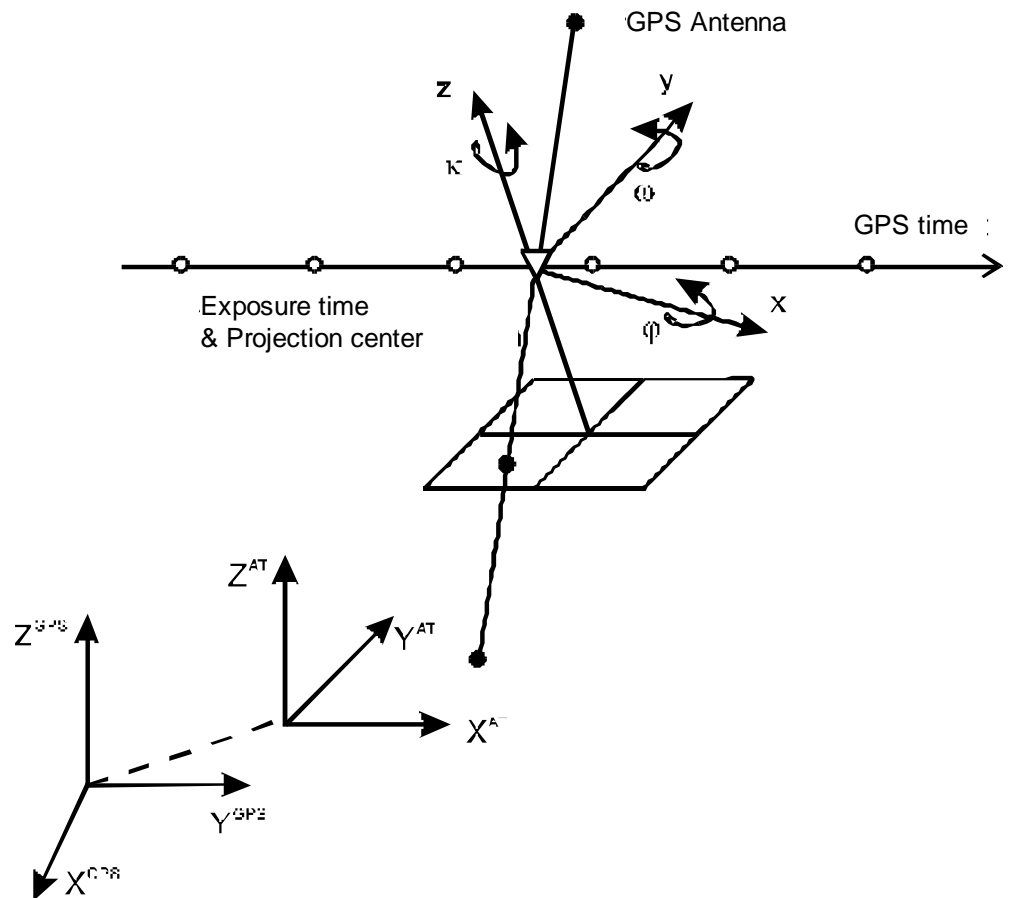


- application for **photogrammetric application**
- **sophisticated feature**: subsequent processing with  
  
GEONAP-K package for GPS data and  
BINGO for combined adjustment
- only operationally applied **rigorous GPS modeling  
in block adjustment**
- also termed CPAS (Combined Phase Ambiguity Solution)

# Relating GPS and Aerial Triangulation



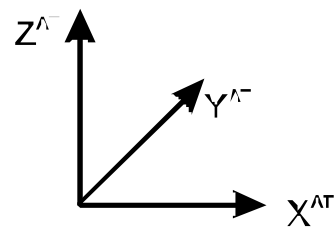
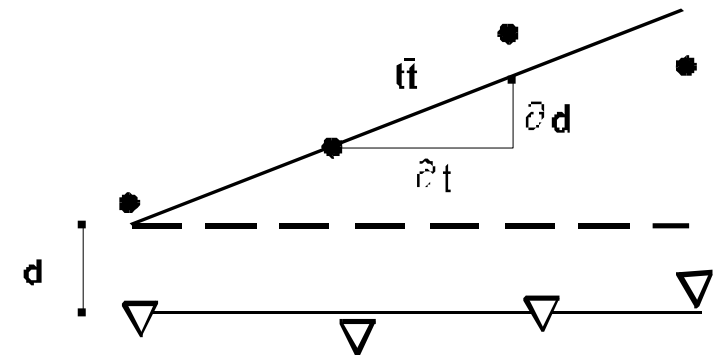
- pre-requisite: unchanged or known conditions of antenna/camera for the complete photo-flight
  - **identical reference point** by orientated vector antenna/camera
  - **identical reference time** by interpolation of GPS coordinates or synchronization of GPS and camera
  - **identical geodetic datum** by datum transformation and/or adjustment



# Shift- and Drift- Approximation in GPS/Block Adjustment



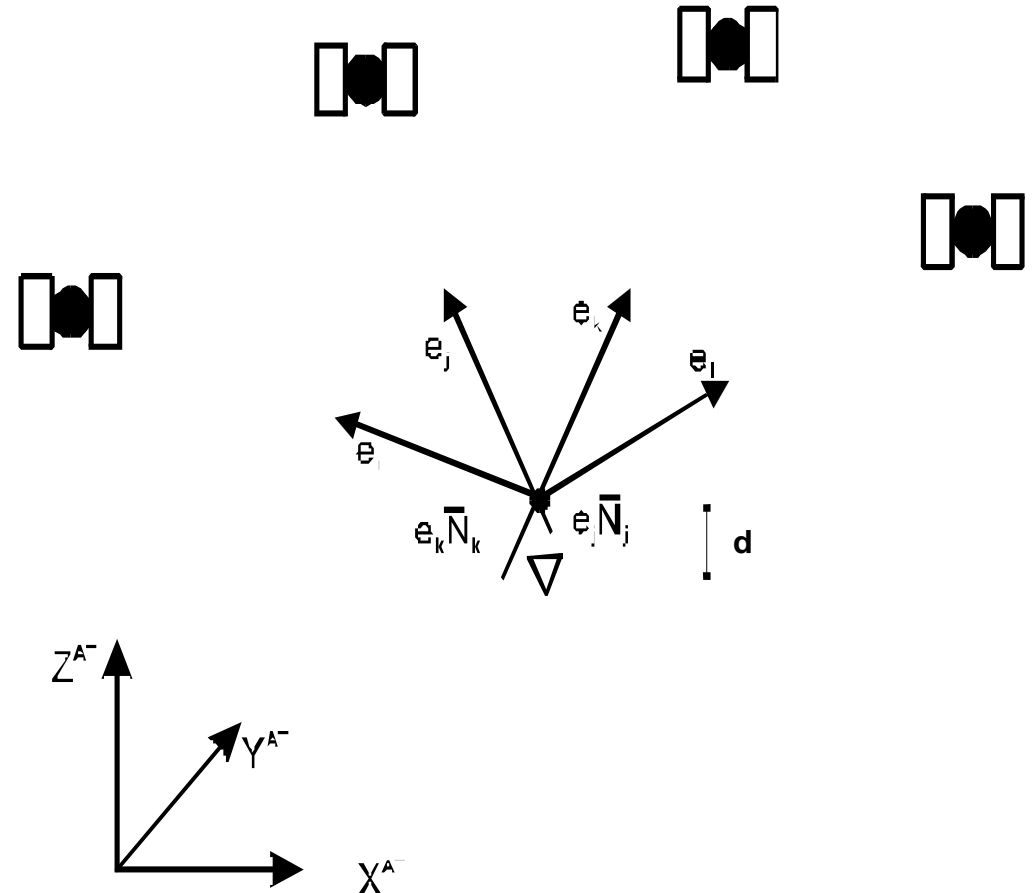
- projection center • GPS and  $\nabla$  AT
- functioning of the shift- and drift-parameter
  - translation and time/strip dependent corrections
    - generally for every strip
    - individual for coordinate components
    - no relationship between strips by GPS
    - considers no changes of satellite constellation
    - approximation of actual GPS model



# Rigorous GPS Model in GPS/Block Adjustment



- projection center • GPS and  $\nabla$  AT
- functioning of rigorous GPS modeling
  - unreliable ambiguities / constellation changes
    - considers actual GPS satellite constellation
    - estimates range/position correction
    - keeps geometric GPS relationship
    - reduces correlation with other parameter



# Comparison of the Mathematical Models



- rigorous GPS model

$$X_{P_i}^{AT} = X_{A_i}^{GPS} + dX_D + (QA^T P)_i * \bar{N}_i + R_i(\phi \omega \kappa) * dX_A$$

- shift and drift approach

$$X_{P_i}^{AT} = X_{A_i}^{GPS} + dX_D + (dSP_i) + R_i(\phi \omega \kappa) * dX_A$$

$X_{P_i}^{AT}$	coordinates of projection center
$X_{A_i}^{GPS}$	(interpolated) coordinates GPS antenna
$dX_D$	datum transformation
$QA^T P_i$	GPS design information
$\bar{N}_i$	ambiguity/range term
$R_i(\phi \omega \kappa)$	rotation matrix from block adjustment/IMU
$dX_A$	vector GPS antenna/projection center
$dSP_i$	shift & drift parameter term
$i$	photo i

# Interface between GPS and Block Adjustment



- complete design-information accessible by elevation  $e$  and azimuth  $a$  of the GPS satellites

$$A_j = \begin{bmatrix} a_1 \\ a_2 \\ \dots \\ a_k \end{bmatrix} \quad a_k^T = \begin{bmatrix} e_x \\ e_y \\ e_z \\ c_0 dt \end{bmatrix} = \begin{bmatrix} -\cos e \cos \alpha \\ -\cos e \sin \alpha \\ -\sin e \\ 1 \end{bmatrix}$$

- book keeping of GPS ambiguities  $N$  (wavelength  $\lambda$ )

$$\bar{N}_i = \begin{bmatrix} \bar{N}_1 \\ \bar{N}_2 \\ \dots \\ \bar{N}_k \end{bmatrix} = \lambda \cdot \begin{bmatrix} N_1 \\ N_2 \\ \dots \\ N_k \end{bmatrix} \quad N_k = \begin{cases} 0 & \text{reliable fixed} \\ 1 & \text{not reliable fixed} \end{cases}$$

- estimation of coordinate correction in combined adjustment

$$dX_i = (QA^T P_i) * \bar{N}_i$$



# Benefits of Rigorous Modeling of GPS

---

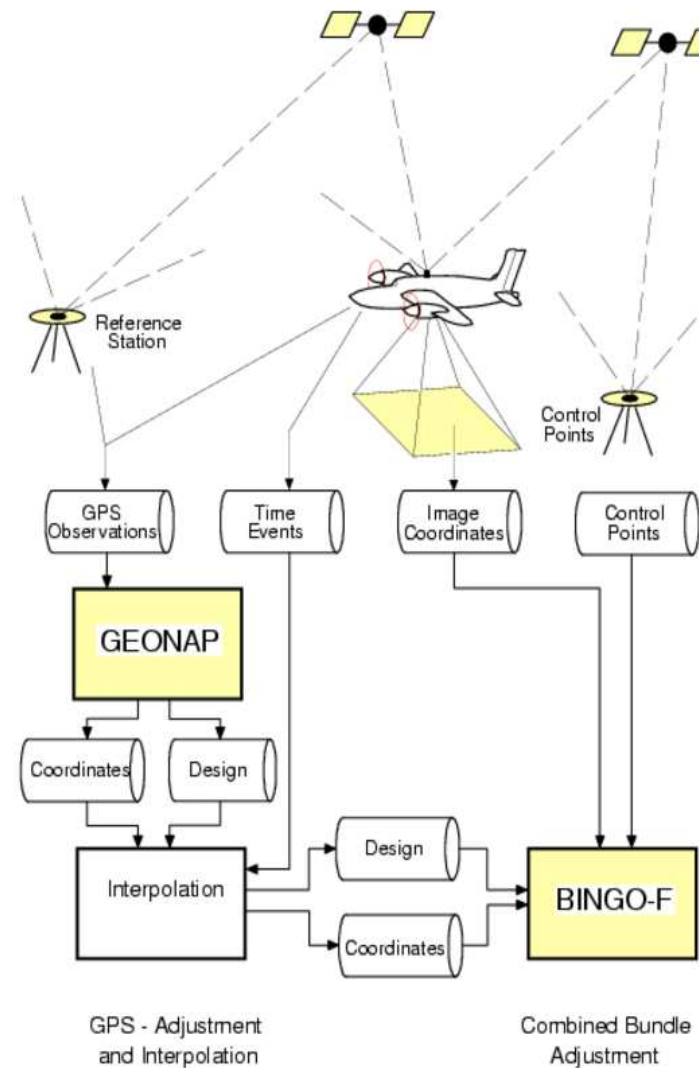


- correct modeling of all GPS errors
- independent of strips/block
- considers the actual GPS model
- considers time dependent effects and GPS constellation changes
- reduced number of unknowns
- relative accuracy of GPS coordinates is maintained (for all strips and complete block)
- no crossing strips required
- separation of systematic GPS errors from
  - e.g. datum parameters
  - additional parameters e.g. interior orientation
- reduction of ground control points and side lap possible
- cost reduction feasible

# Data Flow GEONAP/BINGO



- standardized precise kinematic GPS processing
- provides trajectory of kinematic GPS antenna
- provide design information and ambiguity status for every event
- not necessary to solve all ambiguities
- coordinate transformation
- complete information to rigorously model GPS in block adjustment



# Summary

---



- precise kinematic GPS processing revisited
  - all major error components can be corrected or modeled
  - advantages of multiple reference stations
- rigorous GPS modeling in combined GPS/block adjustment revisited
  - uses the actual GPS satellite geometry
  - advantages and benefits of approach
    - keeps geometric GPS relationship / strengthening of geometry in block adjustment
    - correct functional GPS model / reduced correlation with other parameter
- operational procedure with GEONAP-K and BINGO since 1996



---

for publications on the presented topic refer also to

[www.geopp.com](http://www.geopp.com)

or directly to

[http://www.geopp.com/publications/english/lit\\_e.htm](http://www.geopp.com/publications/english/lit_e.htm)