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Network−Based Techniques for RTK Applications

Gerhard Wübbena, Andreas Bagge, Martin Schmitz
Geo++®, Gesellschaft für satellitengestützte geodätische und navigatorische Technologien mbH
D−30827 Garbsen, Germany
www.geopp.de

BIOGRAPHY

Dr. Gerhard Wübbena received his degrees in Geodesy from the Universität Hannover. He has worked in the field of GPS since 1983. In 1990 he founded the company Geo++®, which develops satellite navigation and positioning software and systems. Among these are the post−processing system GEONAP and the real−time system GNSMART.

Andreas Bagge got his Dipl.−Ing. in Geodesy from the Universität Hannover. Since 1986 he works in the field of GPS, first at the Universität Hannover, and now for several years at Geo++®.

Dr. Martin Schmitz received his degrees in Geodesy from the Universität Hannover. He has been working in the field of GPS for the industry and as a research fellow at the Universität Hannover since 1991.

All authors are currently employed at Geo++®. They are concerned in research and development in satellite positioning. Current projects focus on highly precise RTK phase positioning (GNSMART), GPS station calibration projects and real−time deformation monitoring systems.

ABSTRACT

The accuracy of today's RTK is limited by the distance dependent errors from orbit, ionosphere and troposphere as well as station dependent influences like multipath and antenna phase center variations. The basic idea of Geo++® GNSMART (GNSS − State Monitoring And Representation Technique) is to analyze the data from a reference station network to estimate and represent the state of individual components of the GPS error budget in real−time.

All stations of a network are processed simultaneously for best estimation of global parameters and to increase the reliability of the results. The complete state can normally not be used by the rover directly. Therefore GNSMART can derive several types of representations from the complete state model, adequate for special transmission or rover requirements to reduce the GNSS error budget significantly. The implementation was operable before the current solar activity maximum, and is currently installed on many reference stations around the world under different ionospheric conditions. Recent results show the capabilities of GNSMART. Horizontal accuracy of 1 centimeter can be achieved with initialization times of 30 seconds, often even within 10 seconds over distances of more than 30 kilometer.

INTRODUCTION

Everybody is in the know of the burden to operate a local reference station in the field, in addition to relate the coordinates to the actual local datum. A now commonly accepted remedy is the use of permanently working reference station networks. Benefit is the general increase in reliable RTK (real−time kinematic) positioning, but also over medium distances and under unfavorable atmospheric conditions. The strong disturbances experienced in the culminating solar cycle 23 finally made the need for new strategies in real−time multi−station networks obvious. However, there is a worldwide discussion on these topics, which basically originates from the fact of missing standards. Since the beginning of the 1990's, Geo++® is investigating and developing software in the field of reference station networks (i.e. Wübbena, Bagge 1995, Wübbena et al. 1995). The goal of all research is to predict and represent the complete state of all physical parameters and to enable a rover system to apply autonomously the state information or a derived representation.

UNDIFFERENCED PARAMTER ESTIMATION CONCEPT

Generally, linear combinations of GPS observables are used to eliminate errors. For instance, the so−called double difference GPS observable is the pseudorange difference from two stations to two satellites, which eliminates the receiver and satellite clock biases. It furthermore reduces the effect of highly correlated error terms.

Our approach uses undifferenced observables. Therefore, it is necessary to model and estimate all error components including the clock errors. The gain of information through these models in GPS processing with undifferenced observables will give improved results. It is more rigorous compared to the double differences, where correlations in multi−station applications are typically neglected in the adjustment.

A GPS processing using undifferenced observables can easily be extended for additional unknowns and models. It is even in question, if a double difference software might be able to estimate some parameters of interest. Differencing eliminates error terms but also information. Differenced observables are no longer single station related but vector related. Absolute information in the measurement is differenced out, i.e. differenced observables are not directly usable for absolute positioning.

THE OBSERVATION EQUATION

The observation equation for a pseudorange PR derived from carrier phase measurements is a non−linear function of the geometric range $\left|\vec{\boldsymbol{R}}_i^k\right|$

$$\vec{\boldsymbol{R}}_i^k = \vec{\boldsymbol{X}}^k - \vec{\boldsymbol{X}}_i \qquad (1)$$

between the assumed (broadcasted) antenna phase center of satellite k and receiver i, the ambiguity term $N_{s,i}^k$, several biases, and random measurement errors $\epsilon_{s,i}^k$:

$$PR_{s,i}^k = \left|\vec{\boldsymbol{R}}_i^k\right| + \lambda_s N_{s,i}^k + \delta B_{s,i}^k + \epsilon_{s,i}^k \qquad (2)$$

This pseudorange equation is set up for each signal s that is transmitted by a radio navigation satellite. In case GLONASS is employed, it will be necessary to distinguish the frequencies of the carrier phase signals by using the index f. Different carrier wavelengths λ_s^f for each satellite need then to be introduced. All error terms in (2) and in the subsequent equations are expressed as range errors. Because all quantities are time dependent, a corresponding index is not explicitly applied.

In order to not overload the equations, we assume that the pseudorange in (2) is already corrected for standard portions like broadcasted satellite clock errors, relativistic corrections, tropospheric delay model correction etc. The biases explained in the following therefore describe the remaining model biases for the corresponding terms.

The bias term $\delta B_{s,i}^k$ comprises all clock related errors $\delta C_{s,i}^k$, and systematic influences that will be divided for further analysis in distance dependent errors $\delta D_{s,i}^k$ and mainly station dependent errors $\delta S_{s,i}^k$:

$$\delta B_{s,i}^k = \delta C_{s,i}^k + \delta D_{s,i}^k + \delta S_{s,i}^k \qquad (3)$$

The signal transmission time at the satellite and the signal reception time at the receiver are distorted by clock errors δt_i , δt^k and signal delays $\delta d_{s,i}$, δd_s^k in the hardware of the satellite and the receiver, respectively:

$$\delta C_{s,i}^k = \delta t_i + \delta d_{s,i} - \delta t^k - \delta d_s^k = \delta C_{s,i} - \delta C_s^k \qquad (4)$$

The orbit error vector $\delta \vec{\boldsymbol{o}}^k$ together with signal propagation changes caused by ionosphere $\delta I_{s,i}^k$ and troposphere δT_i^k make up the distance dependent biases:

$$\delta D_{s,i}^k = \frac{\vec{\boldsymbol{R}}_i^k}{\left|\vec{\boldsymbol{R}}_i^k\right|} \delta \vec{\boldsymbol{o}}^k + \delta I_{s,i}^k + \delta T_i^k \qquad (5)$$

The error term $\delta D_{s,i}^k$ cancels out only for short baselines. However, the estimation of these spatial and temporal correlated errors is the key issue for precise real−time positioning. The successful modeling in a reference station network improves the ambiguity fixing with respect to the reduction of the time to fix ambiguity (TTFA), and increases reliability significantly. Positioning with cm−level accuracy becomes practicable also over longer inter−station distances. With the state space approach, a suitable estimation methodology will be proposed in the next sections.

Phase center variations (PCV) of the receiver antenna $\delta A^k_{f,i}$ and multipath $\delta M^k_{s,i}$ are station dependent errors. Just for completeness, also phase center variations $\delta E^k_{f,i}$ and multipath $\delta W^k_{s,i}$ at the satellite antenna are accounted for in the error term $\delta S^k_{s,i}$, although these are not yet considered in practice:

$$\delta S^k_{s,i} = \delta A^k_{f,i} + \delta M^k_{s,i} + \delta E^k_{f,i} + \delta W^k_{s,i} \tag{6}$$

The characteristic of the station dependent components of the error term is that they are uncorrelated between stations. Therefore, the components have to be reduced, corrected or neglected.

Efficient procedures for absolute antenna calibration have already been developed by Wübbena et al. (1996, 2000). Absolute calibration of antennas can compensate errors induced by use of different antenna types, and different antenna orientations. The characteristic of satellite PCV is currently investigated (Mader, Czopek 2001). Methods to determine a site specific multipath pattern are under investigation (Böder et al. 2001), which take advantage of the fixed reflector to antenna geometry at reference stations and of the daily repeatability of multipath. An alternative or additional approach uses the state space estimation within the reference network to average out the remaining multipath since it is uncorrelated between stations.

THE GOAL OF RTK NETWORKS

Being in the field, receiving all necessary correction data via a communication link to determine the absolute position of a GNSS rover station at any location within a few seconds and with millimeter precision is the ideal situation for a GNSS user. Developments of software and algorithms for RTK networks show the way into this direction.

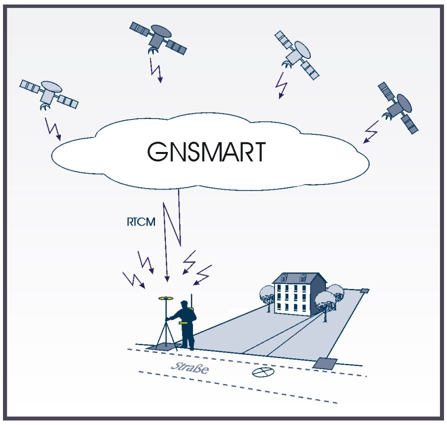

Fig. 1: Millimeter Precision for a GNSS rover with ultimate correction data provided by GNSMART

OBSERVATION SPACE AND STATE SPACE

In post−processing applications, measurements of two or more GPS receivers are processed simultaneously. In real−time differential applications, on the contrary, corrections are derived at reference stations and transmitted to the user. The corrections can be either computed in the position domain or in the observation domain. The position corrections turned out to be impracticable mainly due to the fact that identical satellites must be observed at the reference and the user site.

Pseudorange and carrier phase corrections offer much more flexibility for differential positioning and hence they are the basic parameters in the commonly applied RTCM standard. It will be shown later that this type of corrections are equivalent to single or double difference processing of raw measurements. In either way, however, all errors determined at a reference station are lumped to one parameter describing the total influence on the observation. I.e. the corrections are given in the observation space domain. This means in practice that the user has to be aware of the principle problems related to this approach. In standard RTK applications for instance, the distance to the reference station is one performance limiting factor. Another problem is, that reference station specific errors like PCV and multipath may be inherent in the corrections.

Based on the already mentioned undifferenced parameter estimation concept, differential GPS positioning in the state space domain is proposed as an alternative concept. Instead of generating just one lump sum parameter, the state of each error component is determined from observations of a network of reference stations.

THE IDEAL SOLUTION

The ideal situation for a GNSS rover user in the field would be to get the information about the state of all the external error terms affecting his measurement with millimeter accuracy from one or more service providers or, more convenient, from the satellites itself.

However, the models currently applied are not sufficient to describe the complete state with millimeter accuracy in a global sense. Effects with high spatial correlation, like satellite antenna phase center variations or multipath at the satellite are not known very well and thus limit the precision of the models. Such model deficiencies, however, can be compensated by other state parameters in regional networks.

Another topic on the way to such an ideal solution is standardization. The state parameters of the satellites and the atmosphere need to be described in an international accepted standard. There are currently discussions on network RTK within the RTCM group. Even with simple models related to the observation space the discussion process is slow (Euler et al. 2001).

The advantages of state space representation are found in several aspects. State space representation of the error terms of satellites and upper atmosphere are no longer related to a specific reference station (including a virtual reference station) or group of reference stations. For instance, the state of satellite dependent error components may be derived from different observations than the atmospheric errors. The station dependent tropospheric delay parameters can be derived from measurements to other satellites or even from other devices/techniques like water vapor radiometers. Reference station dependent errors can be reduced if there is enough redundancy in a network.

Within an RTK network the errors affecting a rovers measurements need to be predicted or interpolated from the reference network.

The prediction or interpolation model for state parameters can be based on the knowledge about the physical behavior of the respective parameters. For instance the function for the influence of orbital errors is purely deterministic. The prediction of tropospheric errors, however, may be much more complex and varying with time. Additional information about the actual meteorological situation could be used to improve the tropospheric prediction.

For the observation space an interpolation of the combined effects has to be done. Only arbitrary functional models like polynomials and/or stochastic interpolation are efficient. This makes an optimum solution impossible.

A network running in state space mode will be able to provide information in state space as well as in observation space domain. Failures of individual reference stations will generally not affect a network in state space mode. However, a network in pure observation space mode may fail in the respective areas.

A reference network in a multi−station mode using complete state space modeling of observations results in better performance than simple approaches in observation space with only a few stations involved (typically three). This is due to the fact, that the redundancy in the first approach is much higher than in the second and the complete model is more resistant against biased estimates. This means that the distance between reference stations may be much larger in state space mode and the performance, for instance, with respect to the time to fix ambiguities and reliability, is better.

Real−time applications require a communication link between a service provider and user. The bandwidth of this link is one important design factor. With state space representation an optimum bandwidth will be possible, since the update rates for the different parameters can be optimized with respect to their physical behavior. Another aspect is the necessity for duplex links in case of the so−called virtual reference station (VRS) approach. With state space information a simplex link is sufficient, which allows broadcasting media to be used for an unlimited number of users.

The importance of state space modeling goes beyond precise positioning of rover receivers in a RTK network. Of equal significance is the operation of a real−time multi−station reference network. The state vector contains all information necessary for monitoring the complex dynamic system, which gives the DGPS service provider the opportunity to optimize the network configuration. In case of irregular conditions of one of the state parameters, warnings can be issued to the users.

EXISTING STATE SPACE APPROACHES

Precise ephemeris routinely used in post−processing are an example of state space parameters. In addition, Zumberge et al. (1997) derived precise satellite clock corrections enabling precise point positioning (PPP) for a single station. This approach has been extended for real−time positioning with better than 2 decimeter accuracy in a global DGPS network (Muellerschoen et al. 2001) by supplying corrections to broadcast ephemeris and clock parameters derived from state space modeling.

For RTK networks Geo++® GNSMART (GNSS − State Monitoring And Representation Technique) uses state space modeling based on carrier phase observables (Wübbena, Willgalis 2001, Wübbena et al. 2001).

CURRENT RTK − TRANSMISSION OF REFERENCE DATA

For the transmission of GPS data from a reference station to a rover station two general kinds of data types are currently used. Referring to the RTCM format these are GPS raw data types 18/19 and GPS correction data types 20/21.

It will be shown, that both data types are equivalent considering a single difference observation equation at the rover site. The single differences are used only for a clear writing. An essential feature of GNSMART is the use of undifferenced observables, which requires and enables the modeling of all components of the error term $\delta B_{s,i}^{k}$ explicitly. Double difference processing can proceed with the combination of single differences.

For types 18/19, a single difference for satellite k can be computed between the reference i and rover station j using observable (2). The Δ sign has been replaced in the notation by ϵ for biases, that cannot be modeled by the rover and are therefore acting as errors:

$$\Delta PR_{s,ij}^{k} = \Delta\left|\vec{\boldsymbol{R}}_{ij}^{k}\right| + \lambda \Delta N_{ij}^{k} + \Delta \delta C_{ij} + \epsilon \delta D_{ij}^{k} + \epsilon \delta S_{ij}^{k} + \epsilon_{ij}^{k} \qquad (7)$$

For types 20/21 the correction term is computed from the actual GPS measurements to a satellite, known reference position and broadcast ephemeris. In order to have small numerical values, the reference station clock $\delta\,\tilde{t}_i^k$ is estimated and used to reduce the clock bias $\delta\,C_{s,i}^k$. Instead of equation (2), the transmitted pseudorange correction $PRC_{s,i}^k$ generated at a reference station reads:

$$PRC_{s,i}^k = PR_{s,i}^k - \left|\vec{\boldsymbol{R}}_i^k\right| - \delta\,\tilde{t}_i^k \tag{8}$$

Substituting equation (2) yields for the correction:

$$PRC_{s,i}^k = \lambda_s\,N_{s,i}^k + \delta\,B_{s,i}^k - \delta\,\tilde{t}_i^k + \epsilon_{s,i}^k \tag{9}$$

I.e. the pseudorange or carrier phase corrections are essentially the sum of all biases in the observation plus the random measurement errors. A constant receiver clock term may be added for all satellites to reduce the size of the numerical values. The correction is ambiguous for carrier phase measurements.

Using the above correction term to generate a corrected pseudorange observation equation $CPR_{s,ij}^k$ for the rover station *j*, yields:

$$CPR_{s,ij}^k = PR_{s,j}^k - PRC_{s,i}^k \tag{10}$$

Introducing equation (2) and (8) with its bias terms and re−arranging the single difference leads to:

$$\begin{aligned} CPR_{ij}^k = {} & \Delta\left|\vec{\boldsymbol{R}}_{ij}^k\right| + \lambda\,\Delta\,N_{ij}^k + \Delta\,\delta\,C_{ij} + \delta\,\tilde{t}_i \\ & + \epsilon\,\delta\,D_{ij}^k + \epsilon\,\delta\,S_{ij}^k + \epsilon_{ij}^k \end{aligned} \tag{11}$$

The comparison with (7) shows, that corrections are related to single differences, except for the receiver clock bias term. However, this cancels out through double difference processing. It is essential, that there is an identical error term

$$\epsilon_{\Sigma,ij}^k = \epsilon\,\delta\,D_{ij}^k + \epsilon\,\delta\,S_{ij}^k + \epsilon_{ij}^k \tag{12}$$

in equation (7) and (11). There is only a constant difference in the receiver clock of the reference station, which does not have any effect on the standard double difference processing. Hence, the RTK application performance is independent of the actual format used to transport reference data to a rover.

RTK NETWORKS

In single reference station RTK, there is no information on the individual error term $\epsilon_{\Sigma,ij}^k$, which is then normally neglected. However, especially the distance dependent biases introduce RTK positioning errors. The main focus of a network processing is the modeling and representation of the error components of $\delta\,\hat{D}_{ij}^k$, which can be predicted for any rover position to provide better, reliable and faster positioning.

In the following we will present the network information as additional terms added to the basic corrected pseudorange as derived from type 20/21 messages from one reference station. This approach is currently used in real implementations due to a number of practical reasons. First of all the basic correction messages cover the major part of the error components. Especially the fast varying satellite clock errors. The update rate for this part of the corrections is typically 1 Hz or more. The distance dependent errors have much lower dynamics, i.e. they can be updated at a lower rate. The latency of the basic corrections should be small, whereas the latency of the additional network information can be higher.

The corrected pseudorange observation equation from a reference station network using RTCM types 20/21 data currently is similar to:

$$\begin{aligned} CPR_{ij}^k = {} & \Delta\left|\vec{\boldsymbol{R}}_{ij}^k\right| + \lambda\,\Delta\,N_{ij}^k + \Delta\,\delta\,C_{ij}^k - \delta\,\tilde{t}_i^k + \Delta\,\delta\,\hat{D}_{ij}^k \\ & + \epsilon\,\delta\,S_{ij}^k + \epsilon\,\delta\,\bar{P}_n^k + \epsilon_{ij}^k \end{aligned} \tag{13}$$

The differences to equation (11) are the correction term for the distance dependent errors and the network dependent prediction error $\epsilon\,\delta\,\bar{P}_n^k$.

The network prediction error (network representation error) is a complex function of the number of reference stations *n*, their spatial distribution and other factors. It includes the model error of the prediction or interpolation as well as an indirect effect of all participating reference station dependent errors. The rover position relative to the reference stations also affects the magnitude of this residual error term.

One often formulated requirement is, that a rover close to one reference station position should not be affected by the additional network information, i.e. it should get the basic corrections from that particular reference station. To fulfill this requirement, the network cannot correct for station dependent errors of the reference stations. However, in a multi−station network it is possible to estimate station dependent errors and to derive a correction term. Since the station dependent errors are not spatially correlated, it would be worthwhile to correct for them.

The correction data $CPR_{s,ij}^k$ are computed from actual data of the reference station *i*. Therefore the station dependent error term $\epsilon\,\delta\,S_{ij}^k$ of this particular station *i* and the rover *j* is completely part of the correction signal (e.g. multipath effect).

For the use at a rover station, an adequate representation of the correction data $\Delta\,\delta\,\hat{D}_{ij}^k$ is required. Up to now, reference station networks consist often only of a few stations. This circumstance limits the development of suitable models for network RTK as well as the progress of standards for carrier phase based differential positioning. The mode of operation and the capabilities of software resemble in most cases RTK algorithms, which are not suitable for medium range RTK, and do not take full advantage of the information of a network.

The rover manufacturers currently also demand for more information on the provided reference data, which requires new features of the current standards (Euler et al. 2001).

RTK NETWORKS − FKP MODE

One way of representing the additional corrections for the distance dependent errors is a polynomial parametrization to describe the influence for any rover position in a certain area. Depending on the temporal and spatial variation the orders of the representation must be defined. The RTCM standard currently limits the correction data to be formulated in the observation space, which means, that modified GPS observable must be used.

The area correction parameters (commonly called FKP), are the most flexible and suitable way to represent the state. FKP can be assumed for this discussion as a representation of the full state space information. FKP are more or less simplified to reduce the required bandwidth for transmission or the complexity to apply it at the rover. The state has to be transferred to the observation space, because most rover systems are currently not capable to handle any state space information. The FKP allow the prediction of the distance dependent error term for the approximately known rover position:

$$\Delta\delta\hat{D}_{ij}^{k}=f(FKP_i^k,\Delta\varphi_{ij},\Delta\lambda_{ij},\Delta h_{ij}) \tag{14}$$

This can be done independently from the network processing as only the rover coordinates and satellite information are required.

It is a major advantage, that FKP can be distributed by broadcast media, which is requested by most service providers. The FKP do also not contain absolute tropospheric information, but gradients of the troposphere. The tropospheric effect for a reference station can therefore be figured out and applied correctly to the data by the rover.

The dimensions of networks and the coverage of distribution media often make a linear FKP representation sufficient. The coverage of a linear FKP model is then centered to a real reference station, and the FKP describe the horizontal gradients for the geometric and ionospheric signal components in the observation space (Fig. 2).

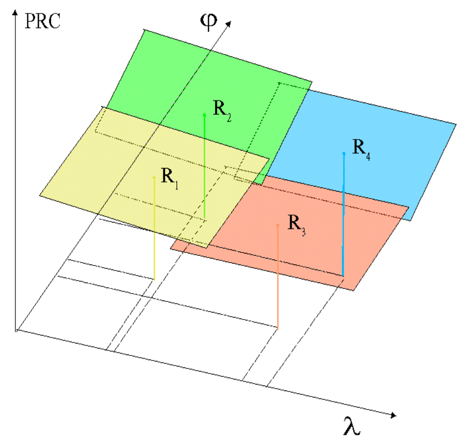

Fig. 2: Linear FKP planes for four reference stations

A virtual reference station (VRS) can easily be computed from FKP, because all relevant information is included in the data stream to individualize the corrections for a given position. The individualization can even be computed at the rover, preserving the advantage of broadcast signals.

RTK NETWORKS − VRS MODE

A pre−requisite of the virtual reference station (VRS) concept is the need of a duplex communication link between a node of the reference station network and the rover.

The rover has to transmit its approximate coordinates to the network, which then interpolates from the state information a reference data stream VRS_{ij}^{k} for the given position. The data relates to the observation space:

$$\vec{X}_{VRS}\overset{!}{=}\vec{X}_j \tag{15}$$

$$VRS_{ij}^k=CPR_{ij}^k+f(FKP_i^k,\Delta\varphi_{ij},\Delta\lambda_{ij},\Delta h_{ij}) + \Delta T_{model,ij} \tag{16}$$

Equation (16) contains a tropospheric term $\Delta T_{model,ij}$, which describes the difference between the tropospheric delay model used in the network processing on the original reference station and the virtual reference station.

Due to the RTCM definitions, the reference station may not correct for tropospheric errors. This is in general a reasonable restriction, because it avoids the problem of using inconsistent models for reference station and rover, while the rover is responsible to compute corrections for both sides. This, however, requires the knowledge of the reference station coordinates at the rover. Since the only coordinates the rover knows about are originating from the RTCM data stream, the rover does only know the coordinates of the VRS. Hence, the rover cannot compute the tropospheric correction for the real, but only for the VRS. In consequence, the network has to apply the tropospheric correction between real and virtual reference station. And here there is again the problem of possible inconsistency, if it is done with a different model than the rover applies.

In the VRS concept, the coordinates (in RTCM message type 3) are changed to VRS location, hiding the true reference station completely from the rover. One disadvantage of the VRS concept is, that for a kinematic rover continuously updated approximate coordinates have to be used for the VRS computation (moving reference station). Today, most rover systems cannot handle a kinematic reference station. A system reset is performed, if the VRS coordinates are changing, which will result in frequent initialization of ambiguities. In practice, the VRS position therefore does not change. However, this implies that distance dependent errors will be present in the rovers solution once it starts to move away from the virtual reference.

Typically, some irregular physical effects occur, which can hardly be determined by a reference station network with given station distances. In this context, the reference station network can be considered as a limited number of monitoring stations or sensors with a certain and restricted spatial capability. The errors may arise from local troposphere or turbulent ionospheric conditions. Even if these higher order errors cannot be determined by the reference station network, it is obvious that their magnitude is a function of distance from the next true reference station. Thus, if the rover knows the reference station position(s), it can take into account these higher order errors and improve its own RTK models, e.g. by stochastic ionospheric modeling. If the rover knows only the VRS position, it has no chance to do such kind of improvement.

It should be mentioned that there are different types of VRS depending on the type of networking model. A VRS derived from the observation space (*OSP*−VRS) shows different behavior than a VRS derived from a state space model (*SSP*−VRS). This results from the fact, that a *SSP*−VRS is much less affected by current individual reference station errors than the *OSP*−VRS. Since the state vector is the result of a continuously running filter, the influence of station dependent errors reduces, the more redundancy (number of stations and satellites) is available in the network. A similar filtering in the observation space can only be done with arbitrary models and is therefore less effective. Especially the non−dispersive part of the signal is much smoother if derived from state information than from the observation state. GNSMART provides *SSP*−VRS.

RTK NETWORKS − STATE SPACE MODEL

The use of the non−dispersive characteristic of some error components is a common procedure to generate an geometric and an ionospheric part of the error budget. The geometric part mainly contains the orbit and tropospheric error components, while the dispersive part is due to the ionospheric propagation delay in the atmosphere. Both parts are correlated through signal delays and clocks.

The state space model, however, performs a functional and stochastic separation also of the tropospheric and orbit error component. It is even possible to estimate the discussed error terms of higher order in the state space model of a reference station network.

In principle, it should be possible to use state vector components estimated in different networks. Parameters with global character, like satellite orbits and clocks, could be determined in a global network. The regional trend of the ionospheric delay is best estimated in regional networks, and small scale networks are needed for modeling local ionospheric and tropospheric effects. Following this strategy, new concepts regarding distribution and spacing of reference stations are conceivable. Densely populated areas and important economic regions will be covered by a close−meshed reference station network for highest accuracy and reliability in positioning, whereas less important areas are covered by a wide−meshed network of regional or national extension. There is no need that all levels of networks are operated by the same provider. The main problem realizing such an hierarchical approach is to ensure the data consistency, which is difficult because of high correlations between state parameters. The definition of suitable standards is indispensable for such plans.

Assuming that the state parameters are estimated with sufficient accuracy, they are transmitted to the user who can eliminate the corresponding error terms of his observation equation, and derives a precise absolute position. The use of corrections in the state space domain requires again suitable standards, which are currently not available. Instead, the state parameters are reduced to observation domain corrections.

Generally, a rover has not much knowledge about the quality of the received reference data, which would enable autonomous decision for the RTK rover processing. Better concepts are possible, if the actual state information and stochastic behavior of the residual errors are used instead of corrections in the observation space. Actual state information can describe all physical effects at the rover location without even providing observation data of a reference station. Only the transformation of the state into the observation space of the rover is required using standardized models.

STATE SPACE ESTIMATION

Bias	*Functional Model*	*Stochastic Model*
Satellite clock	2nd order polynomial	white noise process
Signal delay (sv)	constant	integrated white noise process
Satellite orbit	cartesian elements	3D Gauss−Markov process
Ionospheric delay	single layer model with polynomial (φ, λ) 1 bias per sv (vertical delay)	3D Gauss−Markov process (1 bias per rcv−sv combination)
Tropospheric delay	modified Hopfield model	2 scaling parameter/station
Receiver clock offset	−	white noise process (1 parameter/epoch)
Signal delay (rcv)	constant	integrated white noise process
Satellite PCV	−	−
Receiver PCV	calibration	−
Multipath (rcv)	elevation dependent weighting	1st order Gauss−Markov process
Measurement noise	−	white noise process
Ambiguity	constant if fixed	−

Table 1: Functional and stochastic description of GPS error sources in GNSMART

The Kalman filter is proofed to be well suited for state estimation and monitoring tasks (Wübbena 1991). The methodology of state space modeling can be applied to real−time carrier phase based positioning in multi−station networks, which was suggested by Wübbena, Bagge (1997).

Fast and reliable ambiguity resolution in the multi−station network requires highly accurate coordinates for the reference stations. This applies to the absolute position in the global reference frame as well as to the relative coordinate vectors between the stations. Assuming that calibrated antennas are used, the ambiguity term $N_{s,i}^k$ has to be split off from the residuals containing multipath and distance dependent errors:

$$N_{s,i}^k = \frac{1}{\lambda}\left(PR_{s,i}^k - \left(\left|\vec{\boldsymbol{R}}_i^k\right| + \delta\, B_{s,i}^k + \epsilon_{s,i}^k\right)\right) \tag{17}$$

Cycle Slips are detected and eliminated in advance in order to keep the ambiguity vector small. With GNSMART, the ambiguities are then estimated in a simultaneous dual−frequency adjustment together with the complete state vector. Because of the implicit ionospheric model, the adjustment will result in an ionospheric free solution, but with the advantage of the low signal noise of the L1 and L2 frequencies.

A Kalman filter is employed to process the dynamic model. The corresponding measurement model has already been introduced previously in (2). A simplified notation for the Kalman filter reads:

$$\boldsymbol{x}_{t+1} = \boldsymbol{T}_t\, \boldsymbol{x}_t + \boldsymbol{C}_t\, \boldsymbol{w}_t \tag{18}$$

$$\boldsymbol{l}_t = \boldsymbol{A}_t\, \boldsymbol{x}_t - \boldsymbol{v}_t \tag{19}$$

Equations (18) and (19) contain the state vector $\boldsymbol{x}_{t+1}$, transition matrix $\boldsymbol{T}_t$, transfer function matrix $\boldsymbol{C}_t$ for the process noise vector $\boldsymbol{w}_t$, as well as the linearized measurement vector $\boldsymbol{l}_t$, design/system matrix $\boldsymbol{A}_t$ and the measurement error vector $\boldsymbol{v}_t$ for an epoch *t*.

The state vector, where all states are written as sub−vectors, is rather complex and therefore only a brief summary of the functional and stochastic models can be given:

$$\boldsymbol{x} = \langle \boldsymbol{X}_i | \boldsymbol{N}_i^k | \delta\, \boldsymbol{t}_i | \delta\, \boldsymbol{t}^k | \delta\, \boldsymbol{o}^k | \delta\, \boldsymbol{T}_i^k | \delta\, \boldsymbol{I}_{s,i}^k | \delta\, \boldsymbol{M}_{s,i}^k \rangle^T \tag{20}$$

Table 1 summarizes the functional and stochastical properties, which are currently used for the state space modeling within GNSMART.

GEO++® GNSMART

Geo++® developed and provides a real−time software package under the conceptional name GNSMART, which stands for GNSS State Monitoring and Representation Technique.

GNSMART has been designed with the following features in view:

- *PC based, multi−tasking operating system*
  The PC platform allows cheap and flexible hardware. GNSMART runs on multitasking operating systems (Windows NT/2000 or IBM OS/2) to allow the integration with commercial or user provided software.
- *Modular system*
  GNSMART is a modular system. It provides all components for a (D)GPS service provider: reference station, communication modules, GPS−multi−station−networking, integrity monitoring, alarm management, data distribution and other special tools. The modules can run on a single computer or distributed over local or wide area computer network.

- *Receiver independence*
  The receiver modules in a GNSMART system allow to connect almost every GPS or GLONASS receiver with RS−232 or TCP/IP interface and programmable input/output. Currently modules for Leica, Ashtech, Javad, Topcon, Trimble and many others are available. Only the very basic data acquisition capabilities without any internal RAM or RTCM options are required, which allows to use cheap sensor hardware.
- *Compatibility to standard formats*
  GNSMART's interfaces to external devices are based on international standards, such as RTCM, NMEA and RINEX. Even the internal communication between the various GNSMART modules is using RTCM format. New features not yet covered by any international standard, e.g. antenna or network parameters, have been developed by Geo++® and proposed for next RTCM versions.
- *Real−time capabilities*
  The parameter estimation modules in GNSMART are using dynamic models in a recursive Kalman filter algorithm. This allows the program to run continuously for unlimited time, without any discontinuities for all estimated parameters
- *Redundancy*
  Since the state information is derived from all stations in the network, a failure of individual stations does not affect the overall performance of the network.
- *High Performance*
  The models are capable to describe all errors of all visible satellites, even down to 0° elevation. This allows to determine the state information for satellites at very low elevation and therefore yields maximum availability for rovers.

Primary products of GNSMART are available from the state vector:

- satellite clocks and signal delays, satellite orbits,
- ionospheric delay (electron content),
- tropospheric delay (atmospheric humidity),
- receiver clocks and signal delays,
- reference station coordinates.

Derived products of GNSMART are:

- state representation through spatial model (FKP),
- state prediction for rover position (VRS),
- state transformation, e.g. precise regional satellite clocks, orbits, total ionospheric electron content, tropospheric water vapor distribution, dynamic reference station movements.

With these capabilities, GNSMART provides the following service areas:

- single reference stations (DGPS, RTK) (RTCM), VRS (DGPS, RTK) − requires duplex communication link to rover,
- FKP transmission (DGPS, RTK) − broadcast communication to rover,
- filter state transmission (broadcast),
- reverse DGPS/RTK (duplex communication, simple rover),
- real−time and post−processing services.

The GNSMART package is running operational in numerous permanent reference station networks worldwide.

EMPIRCAL RESULTS

Empirical results are presented from a network in Japan, which has been installed for a benchmark test of reference station network software packages at the Geographical Survey Institute (GSI) of Japan in March 2001.

The conditions in Japan are very challenging, because large inter−station distances are present and the ionospheric activity in Japan is much higher compared to Europe or North America.

The network consists of five reference stations (Fig. 3), which uses different GPS equipment. The antennas are corrected using absolute PCV type means to achieve homogeneous conditions. The inter−station distance range from 32 to 95 kilometers, while a monitoring/rover station is operated in a distance of 26 to 32 kilometers from the three closest reference stations 2005, 2001 and 2002. The GNSMART network used the VRS mode for the transmission of corrections.

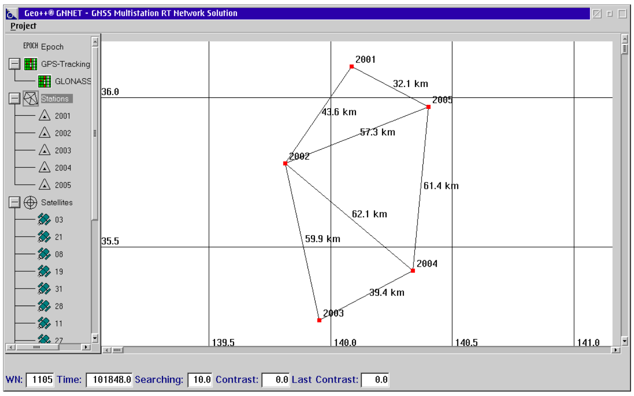

Fig. 3: Screen shot of reference station distribution of GNNET, main module of GNSMART, GSI RTK network, Japan

However, the network has to be considered as a quite "weak" network, because only the minimum number of recommended stations are involved. More stations improve the redundancy and performance of a RTK network, which has been experienced with larger installations in Germany.

A GNNET−RTK rover system operated on the monitor station and performed over a time period of 3 hours automatically ambiguity resets 10 seconds after a successful ambiguity fixing. Hence, an analysis of time to fix ambiguities (TTFA) is possible. Fig. 5 shows the ambiguity initialization times, which do have a mean of

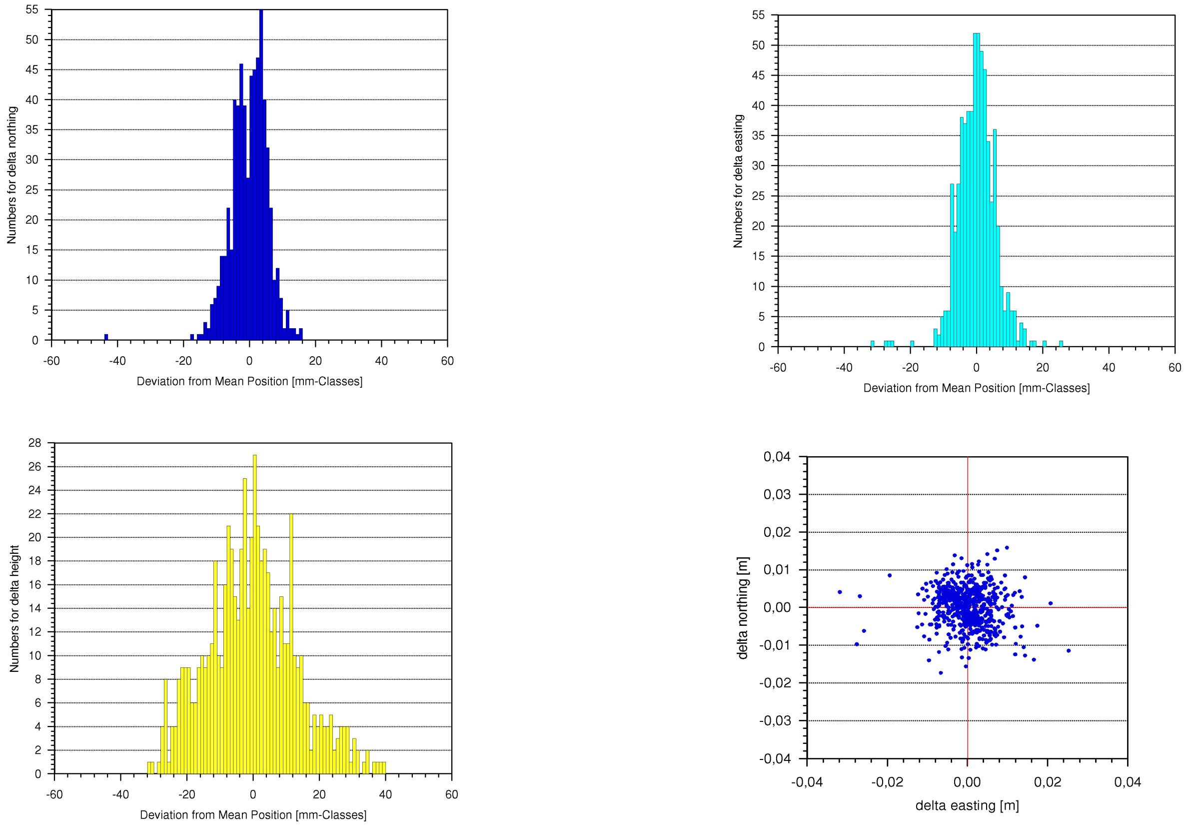

Fig. 4 : Histogram of deviation from the mean position in northing, easting, height (millimeter classes) and horizontal position plot (units are meter), GSI RTK network, Japan

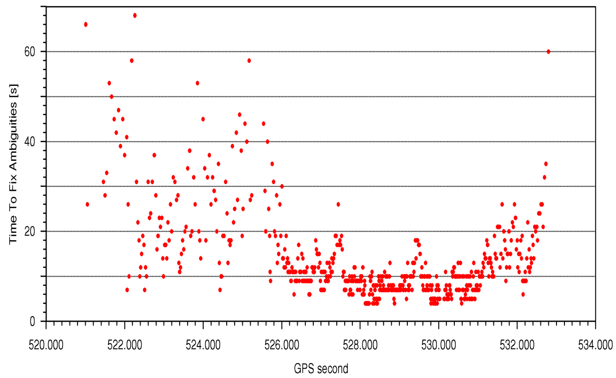

Fig. 5 : Time to Fix Ambiguity (TTFA): mean is 14 seconds, GSI RTK network, Japan

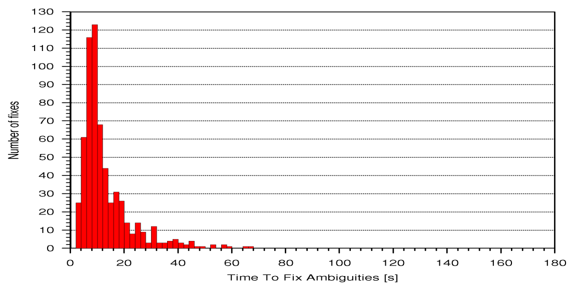

Fig. 6 : Histogram of Time to Fix Ambiguity (TTFA): mean is 14 seconds, GSI RTK network, Japan

14 seconds. The TTFA exceeds in four cases 80 seconds (maximum 160 seconds), which are outside of the plotting range. A histogram of the TTFA reflecting the mean is given in Fig. 6.

The GNNET−RTK system used a simultaneous dual−frequency adjustment of the L1 and L2 frequencies with a stochastic modeling of the remaining ionospheric residuals.

The differences in the three coordinate components to a mean position computed from the approx. 615 RTK position solutions and the horizontal position scatter are displayed in Fig. 4. The standard deviation for the northing, easting and height component are 5.7 millimeter, 5.5 millimeter and 13.5 millimeter, respectively. The overall standard deviation of the 3D position is 15.6 millimeter.

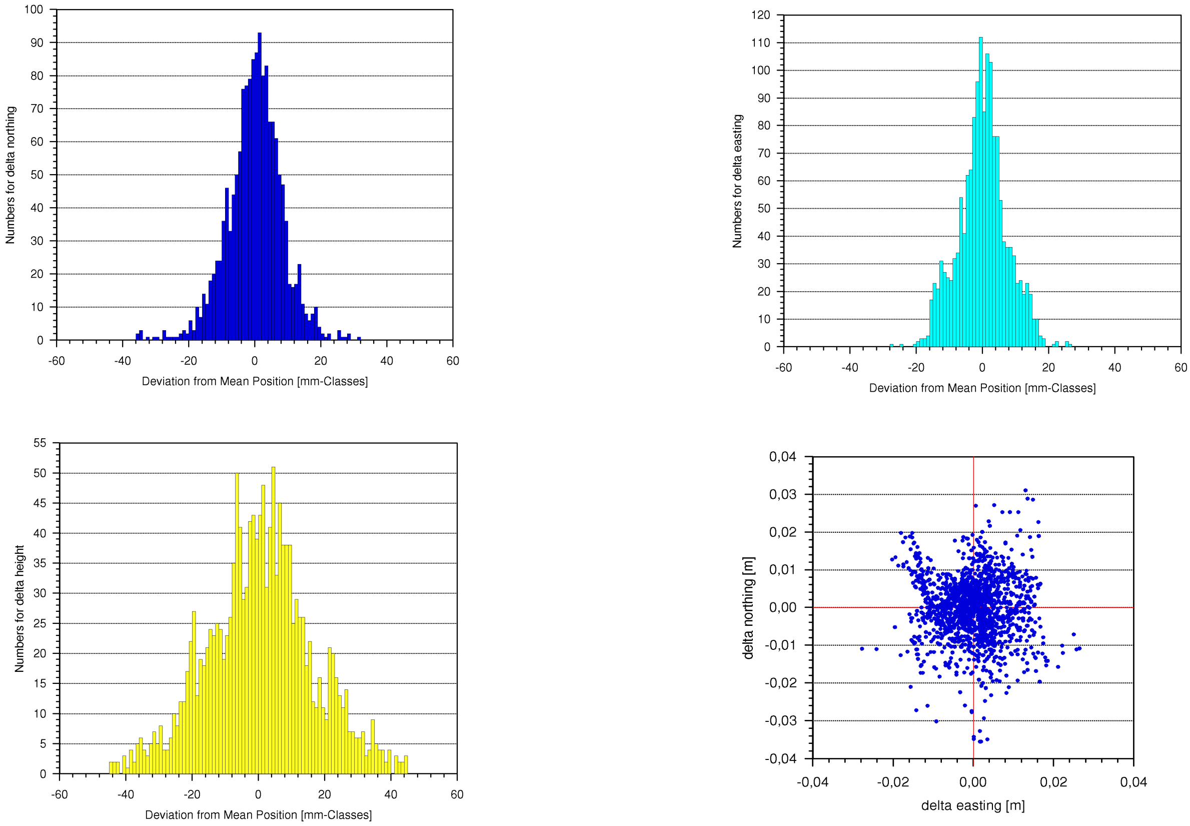

Fig. 7 : Histogram of deviation from known position in northing, easting, height (millimeter classes) and horizontal position plot (units are meter), Brandeburg RTK network, Germany

The real−time data set demonstrates the general performance of a GNSMART network. Accuracies at the cm−level are achieved over long distances and under the very severe ionospheric conditions in Japan.

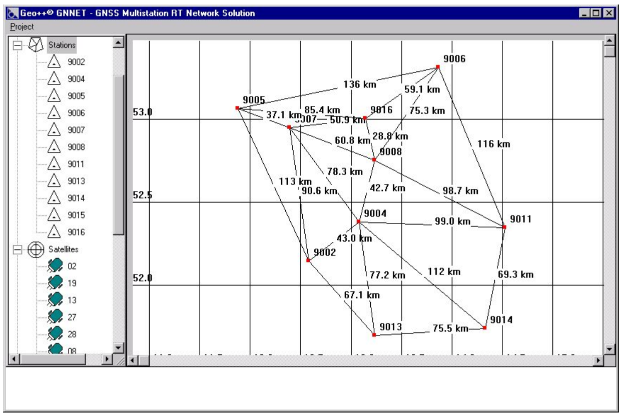

Fig. 8 : Screen shot of reference station distribution of GNNET, main module of GNSMART, Brandenburg RTK network, Germany

The computer resources restrict the actual number of stations to be processed on a single computer, because the number of arithmetic operations increase non−linear with each station. Therefore currently sub−networks are used to distribute the computational burden. The requirements for the sub−network design are mainly to keep the complete information of the overall network and to provide continuity between sub−networks.

Fig. 8 shows a sub−network of the Brandenburg RTK network in Germany. A kind of “hierarchical” network design is used. There is no actual higher order network, but stations from a neighboring networks are used to extend the network dimension. The advantage then is, that long periodical effects of the distant dependent errors can be estimated and can still be used by a rover station.

In the given example, the distances of the surrounding hierarchical station from the core network range from 77 to 116 kilometers. The investigations of sub−networks show, that a sufficient number of station is capable to model all error components within the state space model of GNSMART to an accuracy, which makes ambiguity resolution even over large distances possible.

This has the advantage, that the currently recommended restriction of 70 kilometers station separation can be exceeded. Also a failure of a complete sub−network within a hierarchical network can be compensated, because the long periodical state information is still available from a neighboring sub−network. This is again a significant benefit from the state space model used in GNSMART.

The sub−networks approach allows also the use of independent monitoring stations, which can be used to permanently verify the RTK network performance. The results from a monitor station has been analyzed, which is located in the sparse south−eastern part of the hierarchical network of Fig. 8. The distances to the closest reference stations 9004, 9008 and 9011 are 47 to 58 kilometers. The position of the monitor station has been determined approximately 1544 times with independent ambiguity resolutions during a test period of approximately 13 hours.

The differences in the three coordinate components to the known position are displayed in Fig. 7. The standard deviation for the northing, easting and height component are 8.3 millimeter, 7.4 millimeter and 17.7 millimeter, respectively. The overall standard deviation of the 3D position is 20.9 millimeter.

The Brandenburg RTK network data show the performance of a GNSMART network. Accuracies at the cm−level are achieved for a monitor/rover station in a sparse RTK network with previously not attempted inter−station distances of up to 116 kilometers.

SUMMARY AND CONCLUSIONS

Precise real−time positioning over long distances requires a network of GPS reference stations. Multi−station modeling accounts for the spatial correlated errors, and hence speeds up the ambiguity resolution process. A reference network provides redundancy which ensures higher accuracy and higher reliability for precise real−time positioning.

The rigorous multi−station GNSMART system, which is based on undifferenced observations has been presented with its concepts, implementation and results. The GPS errors components are estimated in a state space model, which also allows a consequent state monitoring. The individual modeling of error components improves the prediction of corrections for GNSS users. Considering the different temporal characteristics of the error components, subsets of corrections can be disseminated at different times in order to reduce bandwidth requirements. For the state representation simplified models have to be used in the observation space since state space corrections are not supported by the current standards. One such representation is the FKP model, a low−order surface model especially suitable for broadcasting PDGPS corrections.

There are several advantages, which suggest to define standards also for transmission of state information. Once the corrections can be broadcast in the state space domain users are enabled to perform absolute positioning with highest accuracy. Relative observations to the nearest reference station are no longer explicitly required. The state space information implicitly provides the user datum, derived from a larger network of reference stations. In the future, the user will just observe all available satellites and will obtain state space corrections from the service provider by means of any communication link.

With the current state space estimation and state representation by FKP or VRS, positioning accuracies of 1 centimeter in less than 30 seconds over 26 to 32 kilometer have been demonstrated by empirical results from an installation of GNSMART in Japan.

Important factors are antenna phase center variations and multipath at the reference station as well as at the rover site. High levels of ionospheric activities and traveling ionospheric disturbances with wavelengths smaller than the reference station spacing also deteriorate the possible accuracy while using GPS error representation in the observation space. However, the use of absolute calibrated antennas is inevitable. The research on spatial variations of multipath at reference station sites needs to be continued.

The state space estimation improves with an increasing number of reference stations, and with a larger coverage area. In principle, the state information can be derived from different networks, but it is also possible to integrate the different, currently independent networks by means of the state space approach. A high redundancy and a homogeneous reference frame over larger areas would be the advantage. The matching of ambiguity levels in different networks is a major part of the ongoing research. Again, for the exchange of consistent state space information an adaptation of standards (models, corrections) is needed.

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