State Space Approach for Precise Real Time Positioning in GPS Reference Networks

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Biography

Dr.-Ing. Gerhard Wübbena received his degrees in Geodesy from the Universität Hannover, Germany. He has been working in the field of GPS since 1983 and has developed the GPS post processing software GEONAP as well as the real time processing software GNSMART. In 1990 he founded his company Geo++, which develops GPS software for static, kinematic and real time kinematic applications.

Stefan Willgalis received his Dipl.-Ing. in Geodesy in 1995 from the Universität Hannover, Germany. For two years he worked at the Technische Universität Braunschweig in the application of GPS for engineering surveys and geodynamic. Since 1998 he is employed as research and teaching associate in satellite positioning at the Institut für Erdmessung, Universität Hannover. His research focus on precise real time GNSS positioning.

Abstract

Networks of GPS reference stations are set up to provide carrier phase corrections for precise real time positioning for an increasing number of applications ranging from legal and geodetic control surveys to geodynamics and hydrography. To exploit the potential of real time multi-station positioning the parameterisation of GPS errors in the state space domain is applied. If the state of each GPS error component can be modelled the estimation of ambiguities will be faster and more reliable even over longer interstation distances. Most important is the individual modelling of the three distance dependent error components orbit, ionosphere, and troposphere. For the state space approach the use of undifferenced observations is recommended, since this allows the estimation of clock parameters as well as the separation of error components. It also makes the modelling of error processes easier compared to approaches based on double differences.

Since the current RTCM standard allows only for carrier phase corrections in the observation domain, the state vector needs to be represented by a simplified model, e.g. Area Correction Parameters (FKP) or Virtual Reference Stations (VRS). Both representations yield comparable results. The use of either methods merely depends on the communication infrastructure between the mobile user and the GPS reference network service provider. For the future it should be accomplished to transmit the state parameters to the mobile user, who is then enabled to operate in an absolute positioning mode.

The behaviour of network coefficients is analysed. Selected positioning results of measurements within the reference network of Lower Saxony, Northern Germany, are presented.

1 Introduction

Precise GPS positioning in real time becomes increasingly important for many geodetic applications. To overcome the limitations of standard RTK systems, reference networks of permanent operating GPS receivers are being installed in many countries worldwide. Operated by state authorities as well as private companies, these active reference stations supply either phase measurements or carrier phase corrections for field users on demand, or transmit the corrections continuously. The recent developments in communication technology, especially of cellular phones, support the latter approach in a favourable way.

Multi station reference networks that have been set up in recent years consist often only of few stations. This circumstance limits the development of suitable models for network RTK just as the current standards for carrier phase based differential positioning. The mode of operation and the reference station software used resemble in most cases RTK algorithms which are not suitable for medium range RTK, and do not take advantage of the information of a multi-station network. The strong disturbances experienced in the culminating solar cycle 23 finally made the need for new strategies in real time multi-station networks obvious.

The federal states of Germany have set up dense networks of permanent GPS reference stations. By 2002, the precise real time positioning service within these networks will be operational in most of the states. With an average station spacing of $50\ km$, about 250 active reference stations will be available in Germany. Many stations in larger clusters provide the necessary redundancy to parameterise GPS errors in the state space domain. The separation of error components, as proposed in the subsequent sections, makes the modelling of the three distance dependent errors, ionosphere, troposphere, and orbit, feasible and hence allows to derive individual corrections for a mobile user.

The state space approach is a means to realize a homogeneous active reference network that will replace geodetic control networks. Field users no longer need to tie their measurements to geodetic markers or to the nearest single reference station. Instead, the network of reference stations realizes the datum by transmitting a state vector of corrections to the user, who then performs precise absolute positioning.

In this investigation, the focus is on distance dependent errors, although the compensation of station dependent errors is of equal importance for real time reference networks. The following presentation, though principally valid for GNSS reference networks, has been limited to GPS for clarity and due to insufficient availability of GLONASS.

2 Parameter Estimation Concept

The observation equation for pseudo ranges $PR^i_{s,j}$ derived from carrier phase measurements is a non-linear function of the geometric range $|\mathbf{R}^i_j|$

$$\mathbf{R}^i_j = \mathbf{X}^i - \mathbf{X}_j \tag{1}$$

between the antenna phase centre of satellite i and receiver j, several biases, random measurement errors $\varepsilon^i_{s,j}$, and the ambiguity term $N^i_{s,j}$:

$$PR^i_{s,j} = |\mathbf{R}^i_j| + \delta B^i_{s,j} + \lambda_s N^i_{s,j} + \varepsilon^i_{s,j} \tag{2}$$

This pseudorange equation is set up for each signal s that is transmitted by a radio navigation satellite. In case GLONASS is employed, it will be necessary to distinguish the frequencies of the carrier phase signals by using the index f. Different carrier wavelengths λ^f_s for each satellite need then to be introduced. All error terms in (2) and in the subsequent three equations are expressed as range errors. Because all quantities are time dependent a corresponding index is not explicitly required.

The bias term $\delta B^i_{s,j}$ comprises all clock related errors $\delta C^i_{s,j}$, and systematic influences that shall be divided for the further analysis in distance dependent errors $\delta D^i_{s,j}$ and station dependent errors $\delta S^i_{s,j}$.

$$\delta B^i_{s,j} = \delta C^i_{s,j} + \delta D^i_{s,j} + \delta S^i_{s,j} \tag{3}$$

The signal transmission time at the satellite and the signal reception time at the receiver are distorted by clock errors ($\delta t^i, \delta t_j$) and signal delays ($\delta d^i_s, \delta d_{s,j}$) in the hardware of the satellite and the receiver respectively:

$$\delta C^i_{s,j} = \delta t^i + \delta d^i_s + \delta t_j + \delta d_{s,j} \tag{4}$$

Signal delays caused by ionosphere ($\delta I^i_{s,j}$) and troposphere (δT^i_j) together with the orbit error vector $\delta \mathbf{o}^i$ make up the distance dependent biases:

$$\delta D^i_{s,j} = \pm \delta I^i_{s,j} + \delta T^i_j + \frac{\mathbf{R}^i_j}{|\mathbf{R}^i_j|}\delta \mathbf{o}^i \tag{5}$$

The estimation of these spatial and temporal correlated errors is the key issue for precise real time positioning. The successful modelling in a reference station network improves the ambiguity fixing with respect to the reduction of the time to fix ambiguity (TTFA), and increases reliability. Positioning with cm-level accuracy becomes then feasible also over longer interstation distances. With the state space approach, a suitable estimation methodology will be proposed in the next section.

Multipath ($\delta M^i_{s,j}$) and phase centre variations (PCV) of the receiver antenna ($\delta A^i_{f,j}$) are station dependent errors. Just for completeness, also phase centre variations ($\delta E^i_{f,j}$) and multipath (δW^i_s) at the satellite antenna have been considered in (6):

$$\delta S^i_{s,j} = \delta A^i_{f,j} + \delta M^i_{s,j} + \delta E^i_{f,j} + \delta W^i_s \tag{6}$$

The characteristic of these error terms is that they are uncorrelated. Only calibration of antennas is a means to compensate errors induced by use of different antenna types, and different antenna orientations. Efficient procedures for absolute antenna calibration have already been developed (Menge *et al.*, 1998). Methods to determine a site specific multipath pattern are under investigation (Böder, 1999) which take advantage of the fixed reflector to antenna geometry at reference stations and of the repeatability of multipath. An alternative approach uses the state space estimation within the reference network to average out the uncorrelated multipath error.

Precise positioning with carrier phases requires the determination of the phase ambiguity in (2). Since the clock parameters, the hardware delays, and the ambiguity term N

are linear dependent, the ambiguity fixing is not a trivial problem. The most common approach to resolve this singularity in the system of observation equations employs the parameter elimination concept. By taking differences between observables, the linear dependent biases are eliminated. The between-receiver single difference eliminates under the assumption of simultaneous measurements the satellite clock bias as well as signal delays in the satellite hardware. If two between-receiver single differences are differenced between two satellites, i.e. a double difference is formed, the receiver clock error and the signal delay in the receiver hardware will be cancelled out.

An alternative approach is based on the parameter estimation concept. All nuisance or bias parameters are modelled and estimated together with the station parameters. The singularity of the observation equations can then be solved by carefully selecting certain ambiguity parameters. It is important to choose the right number of parameters so that the singularity can just be removed, and the ambiguities remain integers. The parameter estimation concept uses undifferenced phase measurements as basic observables, as proposed by Wübbena (1985) and later implemented in the GEONAP post processing software.

The use of undifferenced observations has some advantages especially for precise real time positioning in multi-station networks. The main advantage is the ability to constrain each bias by specific models. The modelling of some errors in the undifferenced formulation is even easier than in the differenced form. Finally, from parameter estimation more insight into each physical process contributing to the complex error budget can be gained.

The parameter estimation allows for the introduction of any appropriate model for clock parameters and hardware delays. The use of different GPS signals enables the differentiation between both errors. After deactivation of S/A, models for atomic clocks can be used again to describe the satellite clock error. Since hardware delays show only minor changes over time, a simple model is suitable to constrain these delays which in turn provides more information compared to differencing methods. The elimination of clock errors and delays by differencing out corresponds theoretically to the model of white noise with infinite variance. The use of an alternative model is impossible.

Another important aspect of using undifferenced observations is that the absolute information in the system of observation equations is retained. Filtering methods can then easily be implemented to describe some errors like ionospheric and tropospheric delays or multipath effects by one- or multidimensional stochastic processes. Differencing of observations, on the contrary, eliminates the absolute information and effects all biases correspondingly. For double differences, the application of stochastic processes is in theory and in practice much more difficult.

With the increasing number of reference stations in real time multi-station networks the selection of linear independent baselines in case of the parameter elimination approach becomes more difficult. In addition, the correlations between double differences need to be considered separately. The parameter estimation offers more flexibility in this respect. The observation equation system is always accompanied by a fully populated covariance matrix. A change in the network configuration caused by the breakdown of one of the reference stations can be compensated without much effort. Different receiver types that are commonly used in DGNSS services are accommodated by introducing suitable models. Even signals of other radio navigation systems like GLONASS, or maybe GALILEO in the future, can be integrated in the system of undifferenced observation equations by modelling the different carrier frequencies for each GLONASS space vehicle separately. For a detailed comparison of the undifferenced and the differenced approach see Grant *et al.* (1990), or de Jonge (1998) for a more recent discussion.

3 State Estimation

In post processing applications, measurements of two or more GPS receivers are processed simultaneously. In real time differential applications, on the contrary, corrections are derived at a reference station and transmitted to the user. The corrections can be either computed in the position domain or in the observation domain. The position corrections turned out to be impracticable since identical satellites must be observed at the reference and the user site. Pseudo range corrections offer much more flexibility for differential positioning and hence they are the basic parameters in the RTCM standard. In either way, however, all errors determined at a reference station are lumped to one parameter.

Based on the parameter estimation concept introduced in the last section, differential GPS positioning in the state space domain is proposed as an alternative concept. Instead of generating just one lumped parameter, the state of each error component is determined from observations of a network of reference stations. In principle, most error terms of the undifferenced observation equation (2) are suitable as state parameters but in practice, easily estimable are only parameters of regional to global significance. Precise ephemeris routinely used in post processing are an example of such state parameters. Zumberge *et al.* (1997) derived in addition precise satellite clock corrections enabling precise point positioning for a single station. This approach has been extended for real time positioning with better than 2 dm accuracy in a global DGPS network (Muellerschoen *et al.*, 2001) by supplying corrections to broadcast ephemeris and clock parameters derived from state space modelling.

The importance of state space modelling goes beyond pre-

Table 1: Functional and stochastic description of GPS error sources

Bias	**Functional Model**	**Stochastic Model**
Satellite clock	2nd order polynomial	white noise process
Signal delay (sv)	constant	integrated white noise process
Satellite orbit	cartesian elements	3D Gauss-Markov process
Ionospheric delay	single layer modell with bilinear polynom (ϕ, λ) 1 bias per sv (vertical delay)	3D Gauss-Markov process (1 bias per rcv-sv combination)
Tropospheric delay	modified Hopfield modell	2 scaling parameter/station
Receiver clock offset	–	white noise process (1 parameter/epoch)
Signal delay (rcv)	constant	integrated white noise process
Satellite PCV	–	–
Receiver PCV	calibration	–
Multipath (rcv)	elevation dependent weighting	1st order Gauss-Markov process
Measurement noise	–	white noise process
Ambiguity of carrier phase measurement	constant if fixed	–

cise positioning of rover receivers in a RTK network. Of equal significance is this approach for the operation of a real time multi-station reference network. The state vector contains all information necessary for monitoring the complex dynamic system, which gives the DGPS service provider the opportunity to optimize the network configuration. In case of irregular conditions of one of the state parameters, warnings can be issued to the users. The advantages of state space modelling for the monitoring and prediction of dynamic systems have been discussed extensively by Heunecke *et al.* (1993) for the case of deformation monitoring in geodetic engineering applications. The Kalman filter proofed to be well suited for the state estimation and monitoring tasks. The same methodology of state space modelling can be applied to real time carrier phase based positioning in multi-station networks, which was suggested by Wübbena and Bagge (1997).

Fast and reliable ambiguity resolution in the multi-station network requires highly accurate coordinates for the reference stations. This applies to the abolute position in the global reference frame as well as to the relative coordinate vectors between the stations. Provided that calibrated antennas are used, the ambiguity term N has to be split off from the residuals containing multipath and distance dependent errors.

$$N^i_{s,j} = \frac{1}{\lambda}(PR^i_{s,j} - (|\mathbf{R}^i_j| + \delta B^i_{s,j} + \varepsilon^i_{s,j})) \qquad (7)$$

Cycle Slips are detected and eliminated in advance in order to keep the ambiguity vector small. The ambiguities are then estimated in a simultaneous dual-frequency adjustment together with the complete state vector. Because of the implicit ionospheric model the adjustment will result in an ionospheric free solution but with the advantage of the low signal noise of the L_1 and L_2 frequencies.

A Kalman filter is employed to process the dynamic model (8). The correspondig measurement model (9) has already been introduced in section 2.

$$\mathbf{x}_{k+1} = \mathbf{T}_k\mathbf{x}_k + \mathbf{C}_k\mathbf{w}_k \qquad (8)$$

$$\mathbf{l}_k = \mathbf{A}_k\mathbf{x}_k - \mathbf{v}_k \qquad (9)$$

The state vector, where all states are written as subvectors, is rather complex and therefore only a brief summary of the functional and stochastic models can be given (Tab. 1).

$$\mathbf{x} = [\mathbf{X}_j|\mathbf{N}^i_j|\delta\mathbf{t}_j|\delta\mathbf{t}^i|\delta\mathbf{o}^i|\delta\mathbf{T}^i_j|\delta\mathbf{I}^i_{s,j}|\delta\mathbf{M}^i_{s,j}]^T \qquad (10)$$

In principle it should be possible to use state vector components estimated in different networks. Parameters with global character, like satellite orbits and clocks, could be determined in a global network. The regional trend of the ionospheric delay is best estimated in regional networks, and small scale networks are needed for modelling local ionospheric and tropospheric effects. Following this strategy, new concepts regarding distribution and spacing of reference stations are conceivable. Densely populated areas and important economic regions will be covered by a close-meshed reference station network for highest accuracy and reliability in positioning, whereas less important areas are covered by a wide-meshed network of regional or national extension. There is no need that all levels of networks are operated by the same provider. The main problem realising such an hierarchical approach is to ensure the data consistency, which is difficult because of

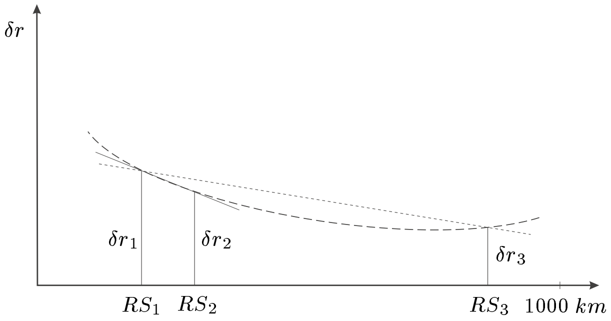

Figure 1: Linear and quadratic interpolation functions

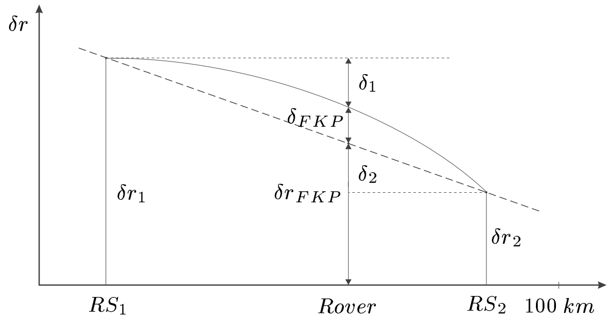

Figure 2: Interpolation of distance dependent errors

high correlations between state parameters. The definition of suitable standards is indispensable for such plans.

Assuming that the state parameters are estimated with sufficient accuracy, they are transmitted to the user who can eliminate the corresponding error terms of his observation equation, and derives a precise absolute position. The use of corrections in the state space domain requires again suitable standards, which are currently not available. Instead, a simplified approach – explained in the next section – is used, in which the state parameters are reduced to observation domain corrections.

4 State Representation

The state estimation, i.e. separation of errors based on observations of a multi-station reference network needs to be distinguished from the state representation. Since the current standards for DGPS positioning do not consider corrections in the state space domain, the representation of the error states by simple mathematical means is required. Low-order surface models, grid based models, or delauny triangulation are some of the possible approaches to describe the spatial variation of distance dependent errors. Independent of the 3D interpolation model, corrections for a user within the reference network can be provided in two different ways. At first, parameters of the interpolation function are transmitted to the user who then computes individual corrections for his approximate location which will improve the observations of the user receiver. Secondly, instead of transmitting parameters to the user, the interpolation function can be used to derive a set of virtual reference station (VRS) observations. For this method, the user is required to send his approximate coordinates to a network computing centre and receives in return the VRS data. With the state space approach, of course, the VRS data can be computed directly, the intermediate step of deriving an interpolation function is not required.

In order to derive a suitable error model, the ambiguity resolution in the reference network is required. After ambiguity fixing, the measured ranges at the reference stations are filtered using the state space model of the complete network. This has an averaging effect on the uncorrelated multipath. The difference between filtered and computed ranges for all fixed satellites yields residuals for the signal s,

$$\delta\hat{r}_s = \hat{R}_{s,meas} - R_{s,comp} \tag{11}$$

which are separated into residuals δr_I of the geometry-free linear combination L_I, and residuals δr_0 of the ionosphere-free linear combination L_0. The geometric residuals can be further divided into orbit and tropospheric residuals. Such distinctions are useful for the further error modelling because of the different dynamics and linearity of the distance dependent error components. Station dependent error components are neglected assuming that reference station antennas as well as user antennas are calibrated, and multipath is averaged out by the state estimation process.

The spatial variations of the residuals are then approximated by a low-order surface model. A bilinear polynomial fits for interstation distances up to 100 km since the spatial correlated errors can be considered as linear as practical experiences show, see e.g. Wanninger (2000). Larger reference station spacings require polynomials of 2nd or even higher order (Fig. 1), depending on the different spatial characteristics of the error sources.

The correction for a user within the network is composed of the range correction $\delta r_{s,j}(t)$ and an additional correction $\delta r_a(t)$ derived from the network. The range corrections require because of their high dynamics update rates of 1 Hz, whereas the network corrections change much less over time. In (12) range corrections from actual measured ranges are used, not from the filtered measurements due to conformity with the RTCM standard.

$$\delta r_{FKP}(t) = \delta r_{s,j}(t) + \delta r_a(t) \tag{12}$$

For the time dependent parameters a of the polynomial (13), which geometrically represent the inclination of a plane, the name area correction parameters, in German abbreviated with FKP, became the practice. They are esti-

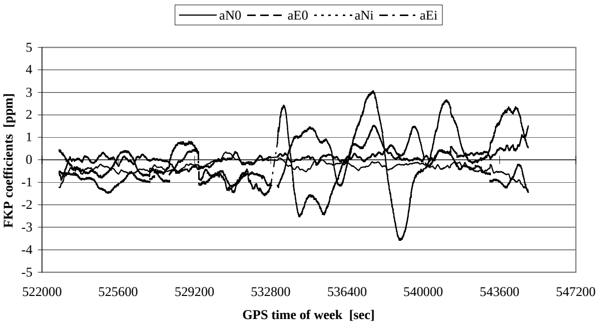

Figure 3: Temporal variations of network coefficients

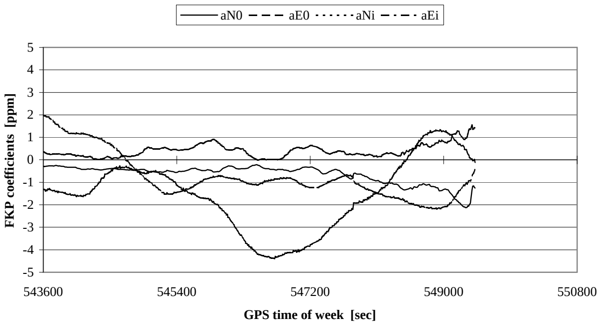

Figure 4: Network coefficients at low elevation

mated in a weighted least squares solution from the residuals (11) at each reference station. Two planes have to be computed for the error components δr_I and δr_0 individually for each satellite, and for each epoch.

$$\delta r_a(t) = a_{\varphi,s}(t)(\varphi - \varphi_0) + a_{\lambda,s}(t)(\lambda - \lambda_0) \quad (13)$$

For the approximate position (φ, λ) of a field user network corrections δr_{FKP} are estimated (Fig. 2) for the ionospheric and the geometric component. Both are substituted back to range residuals for L_1 and $L2$:

$$\delta\hat{r}_1 = \delta\hat{r}_O + \frac{f_2}{f_1}\delta\hat{r}_I \quad (14)$$

$$\delta\hat{r}_2 = \delta\hat{r}_O + \frac{f_1}{f_2}\delta\hat{r}_I \quad (15)$$

The pseudo ranges R, derived from carrier phase measurements of signal s, are finally corrected by

$$\hat{R}_s^i = R_s - \delta\hat{r}_s \quad (16)$$

As shown in Fig. 2, a rover experiences the error δ_1 by using the range corrections δr_1 only. With network corrections δr_{FKP} the much smaller FKP representation error δ_{FKP} remains, which is mainly caused by unmodelled nonlinear effects of the ionosphere. With network corrections, the time for ambiguity fixing is considerably reduced. To compensate the remaining representation error, dual frequency measurements are required.

5 Experimental Results

As a result of investigations since 1992 on the potential of a precise real time positioning service, the State Survey Authority of Lower Saxony, a federal state in northern Germany, now operates 40 GPS reference stations with an average spacing of 50 km. For a set of stations around Hannover the real time multi-station approach previously described is employed. Field users can either use corrections broadcast by VHF radio or virtual reference station data transmitted by cellular phone. In both cases, the states are represented by network coefficients (FKP).

For the investigation a permanent rover, a dual frequency receiver (Ashtech Z-12) with an absolute calibrated antenna (Ashtech Geodetic II), was set up at the Universität Hannover. The RTCM data stream received by VHF radio and the GPS data were stored on a notebook running the GNNET software which computes the network RTK solutions. Because of the nearby reference station Hannover in only 10 km distance the results are not representative for network solutions. Instead an alternative approach was choosen. The RTCM data of nine surrounding reference stations – except Hannover – and the rover were post processed with the real time algorithm. This post processing must not be confused with the conventional GPS post processing, it is instead a real time simulation. The advantages are that different network configurations and different modelling options can be investigated using the same set of data without disturbing the positioning service.

The temporal variations of network coefficients (FKP) are shown in Fig. 3. For a period of six hours between 3 and 10 a.m. local time the four parameters of the bilinear polynomial are plotted for the satellite PRN 05. The variations of the latitude and longitude components of the geometric corrections are below $\pm$ 1 ppm. This is an indication of the high spatial correlation of satellite orbits. The model also describes the tropospheric delay sufficiently, at least for higher elevations. Here the mainly flat topography of Lower Saxony is an advantage. The modelling of the ionospheric delays is compared with that much more difficult. After sunrise at about 6 a.m., the ionospheric components of the network coefficients vary by 6 ppm within just half an hour. Without network corrections, an error of the same size would propagate into the rover position. It is impossible to model such short-term variations by a standard Klobuchar or any other global model.

The tropospheric delay is difficult to model for satellites at low elevation. This causes the geometric network coefficients to increase from ± 1 ppm to almost ± 2 ppm

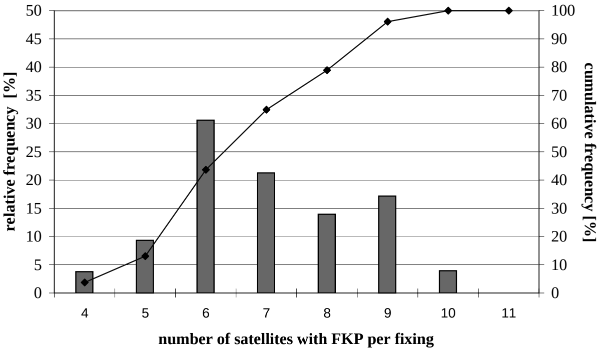

Figure 5: Availability of satellites with FKP

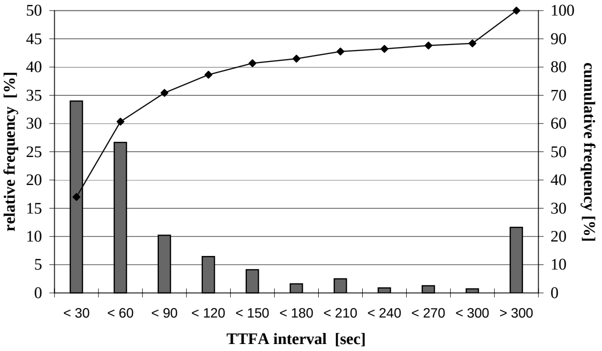

Figure 6: Mean time to fix ambiguities

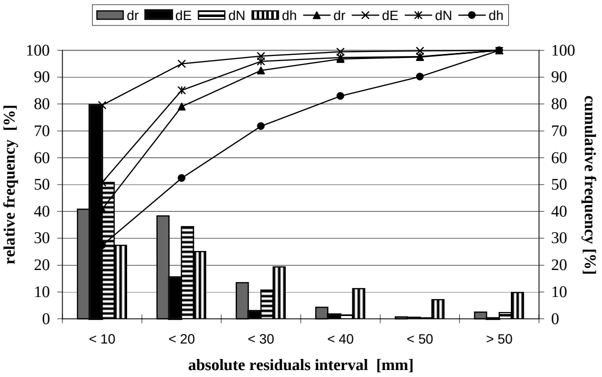

Figure 7: Positioning results for a permanent rover

(Fig. 4) at 10° elevation angle. Since the orbit error remains unchanged, the increase can be attributed solely to tropospheric influences. This is an important information for the state space filter, but currently it is not possible to make this information available for a user. In Fig. 4 the network coefficients for satellite PRN 30 from 9 to 11 a.m. are plotted. It is important to note that the network coefficients do not include a significant multipath signal which would reveal itself by correlations between the geometric and ionospheric residuals.

Besides the characteristics of network coefficients it is important to study for how many satellites FKP are available. A minimum of 5 satellites is necessary for network RTK. For the data set of about 28 hours a total of 560 ambiguity fixings was recorded. A histogram of the number of satellites with FKP for each fixing is plotted in Fig. 5. This and the following figures all present the relative as well as the cumulative frequency for the number of counts in a certain class interval. According to Fig. 5, for 40% of all fixings only up to 6 satellites with FKP were available, although up to 9 satellites were visible above 10° cut-off angle. The remaining satellites had all low elevations. To ensure higher availability the algorithm needs to be refined for fixing satellites between 5° and 10° at reference stations.

For a user, the two most important criteria are the time required for ambiguity fixing and the positioning accuracy. The simultaneous dual-frequency adjustment could finish 70% of all fixings within $90\ sec$, and derived 90% of all solutions in less than $300\ sec$ (Fig. 6). The rover in this case was about $30\ km$ apart from the next reference station.

The residuals between the position solution for each fixing and a reference coordinate for the rover is shown in Fig. 7 for each coordinate component in east (dE), north (dN), height (dh), and the horizontal plane (dr). More than 80% of the residuals are below $2\ cm$, except the height component. Only 10% of the residuals are larger than $3\ cm$. Considering the accuracy requirements of $16\ mm$ for cadastral surveys, precise real time multi-station positioning now has the potential for operational use in cadastral and surveying applications.

6 Conclusions

Precise real time positioning over longer distances requires a network of GPS reference stations. Multi-station modelling accounts for the spatial correlated errors, and hence speeds up the ambiguity resolution process. A reference network provides redundancy which ensures higher accuracy and higher reliability for precise real time positioning.

A rigorous multi-station approach based on undifferenced observations has already been developed and implemented in several countries. The GPS errors are estimated in a state space model, which also allows a consequent state monitoring. The individual modelling of error components improves the prediction of corrections for field users. Considering the different temporal characteristics of the error components, subsets of corrections can be disseminated at different times in order to reduce bandwidth requirements. For the state representation simplified models have to be used since state vector corrections are not supported by the current standards. One such representation is the FKP model, a low-order surface model especially suitable for broadcasting PDGPS corrections.

Once the corrections can be broadcast in the state space domain users are enabled to perform absolute positioning with highest accuracy. Relative observations to the next reference station are no longer explicitly required. The state space information implicitly provides the user datum, derived from a larger network of reference stations. In the future, the user will just observe all available satellites and will obtain state space corrections from the service provider by means of any communication link.

With the current state space estimation and state representation by FKP, positioning accuracies of $1\ cm$ in $1\ min$ over $35\ km$ are in principle feasible. The limiting factors are antenna phase centre variations and multipath at the reference station as well as at the rover site. High levels of ionospheric activities and travelling ionospheric disturbances with wavelengths smaller than the reference station spacing also deteriorate the possible accuracy. Therefore the use of absolute calibrated antennas is inevitable. The research on spatial variations of multipath at reference station sites needs to be continued. With respect to modelling aspects, it is not recommended to increase the reference station spacing beyond $100\ km$.

The state space estimation improves with an increasing number of reference stations, and with a larger coverage area. In principle, the state information can be derived from different networks, but it is also possible to integrate the different, currently independent networks by means of the state space approach. A high redundancy and a homogeneous reference frame over larger areas would be the advantage. The matching the ambiguity levels in different networks is a major part of the ongoing research. For the exchange of consistent state space information an adaptation of standards (models, corrections) is needed.

Acknowledgement

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