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On the modelling of GNSS observations for high−precision position determination

Gerhard Wübbena

Geo++® Gesellschaft für satellitengestützte geodätische und navigatorische Technologien mbH
D−30827 Garbsen, Germany
www.geopp.de

1 Introduction

GPS can now look back on a history of over 20 years. Precise position determination with geodetic accuracy has been carried out successfully since the mid−1980s. Since that time the system as a whole, consisting of the technology, the procedures and the algorithms for modelling GPS observations, has been continuously developed and improved. This is evident to GPS users through, for example, the reduction (from several hours to a few minutes or even seconds) of the observation time required to achieve an accuracy of the order of centimetres for specific applications.

The quality of position fixing using Global Navigation Satellite Systems (GNSS) is defined in terms of criteria for accuracy, availability and reliability. Because of the large number of satellites (currently 27 for GPS), availability is very good even in partly obscured areas. However in more severely shadowed areas it is often necessary to adopt or to integrate other measurement techniques.

To achieve accuracies in the region of centimetres or millimetres there remains a central problem, namely the resolution of carrier phase ambiguities. Methods and algorithms for ambiguity resolution have been very greatly improved. However the availability and reliability of positioning are still restricted by the ambiguity problem.

In addition to static applications, kinematic applications requiring high−precision positioning are coming increasingly to the fore.

Modern communications technologies are continuing to reduce the costs of data transmission, so that real−time applications are increasingly used even over longer distances. While precise real−time applications, such as the so−called real−time kinematic (RTK) techniques, were in the past restricted to short ranges and to two stations (baselines), ever more reference station networks are now being set up, whose data is processed in real time for a variety of applications. The processing of system state information instead of observation corrections represents a new procedure in this context.

These developments lead to new requirements in the modelling and analysis of measurement data.

Starting from the observation equation for code and carrier−phase observations, this paper examines and discusses several aspects of GPS model formation. The presentation is not exhaustive and goes into different aspects in varying levels of detail.

2 The observation equation

The GNSS observation equation for pseudorange measurements using code and carrier−phase signals is:

$$PR^j_{s,i} = \left|\vec{R}^j_i\right| + \delta t^j + \delta d^j_s + \frac{\vec{R}^j_i}{\left|\vec{R}^j_i\right|}\delta\vec{o}^j + \delta S^j_{f,i} \pm \delta I^j_{f,i} + \delta T^j_i + \delta M^j_{s,i} + \delta A^j_{f,i} + \delta d_{s,i} + \delta t_i + \lambda^j_s N^j_{s,i} + \epsilon^j_{s,i}$$

where

i	– indicates the observing station (the receiver),
j	– indicates the observed satellite,
s	– indicates the observed signal (P−code, C/A−code, L1 phase, L2 phase),
f	– indicates the signal frequency (L1, L2),
$\vec{R}_i^j$	– is the range vector from the receiver antenna to the satellite antenna,
δt^j	– is the satellite clock error,
δd_s^j	– is the effect of the transit time delay of the signal *s* in the satellite hardware,
$\delta \vec{o}^j$	– is the orbit error vector of the satellite,
$\delta S_{f,i}^j$	– is the error due to phase centre variations in the satellite antenna,
$\delta I_{f,i}^j$	– is the ionospheric effect on the signal *s* (positive for codes, negative for phases),
δT_i^j	– is the tropospheric effect,
$\delta M_{s,i}^j$	– is the effect of multipath phenomena,
$\delta A_{f,i}^j$	– is the effect of phase centre variations in the receiver antenna,
$\delta d_{s,i}^j$	– is the effect of the transit time delay of the signal in the receiver hardware,
δt_i	– is the effect of receiver clock error,
λ_s^j	– is the wavelength of the signal for the observed GLONASS satellite,
$N_{s,i}^j$	– is the ambiguity of phase measurements (generally only for carrier−phase measurements), and
$\epsilon_{s,i}^j$	– is the random error of measurement.

All quantities are time−dependent to varying extents. The coordinates of the receiving antenna are concealed in the first term on the right hand side of the equation. This is

$$\vec{R}_i^j = \vec{X}_c^j - \vec{X}_i$$

where

$\vec{X}_c^j$	– are the coordinates of the satellite antenna at the instant of transmission of the signal (computed using, for example, the broadcast ephemeris), and
$\vec{X}_i$	– are the coordinates of the receiving antenna at the time of measurement (or reception of the signal).

The accuracy of a position fix using GNSS for kinematic applications may be estimated using the formula

$$\sigma_p = PDOP\,\sigma_l \quad .$$

Here σ_l is the accuracy of a pseudorange measurement including all error components not forming part of the observation equation, PDOP (Position Dilution of Precision) is a factor describing the geometry of the observed satellite constellation, and σ_p is the expected 3D positional accuracy.

3 Undifferenced and differenced observations

High−precision positioning is only possible when the ambiguity in the phase measurement is resolved, that is, the integer value of the unknown *N* is determined. A problem in this connection is that there is a linear relationship between the clock parameters and transit time delays of the hardware as well as the ambiguities themselves. For a given signal *s* these parameters always appear in the observation equation with the same coefficients. The system of observation equations is therefore singular and is not soluble without further information.

In principle various different solutions to this problem can be devised. The procedure which has been most widely used in practice is phase differencing. The so−called “single difference” of measured quantities between two stations (receivers) eliminates the effect of satellite clock error and time delay in the satellite hardware, provided that the measurements are simultaneous. The “double difference”, that is the difference of two single differences between two satellites, further eliminates the receiver clock errors and time delays caused by receiver hardware.

An alternative to differencing techniques for resolving the singularity in the system of equations is the use of estimated clock and ambiguity parameters. Care must be taken here to ensure that the number of parameters estimated exactly eliminates the singularity, and that the integer character of the remaining ambiguities is preserved. With this method the undifferenced observations can be processed, which gives some advantages over differencing procedures.

The differencing process leads to the complete elimination of the clock error and hardware time delays. In theory this is equivalent to modelling or estimating the eliminated parameters, such that an independent unknown is introduced for every measurement epoch. Under the differencing process the clock parameters and transit time delays are therefore in principle modelled as white noise with infinite variance. No other modelling possibilities are available with differencing techniques.

In undifferenced processing, on the other hand, any meaningful model can be applied to the clock parameters and transit time delays. In particular the delays in the hardware vary only very slightly with time, so that an appropriate model leads to a clear gain of information compared with differencing methods. Since selective availability (S/A) was switched off it has again become possible to describe GPS satellite clock errors using an atomic clock model.

The differencing process eliminates not only the clock error and time delays of the hardware, but also operates on all other error sources. The consequence is that all absolute error effects are eliminated and only their differences remain in the system. However, the modelling of such differences becomes markedly harder than the modelling of undifferenced effects. Some error components such as ionospheric or tropospheric transit time or multipath effects can be described in terms of stochastic processes. Individual processes in one dimension or more can then be relatively simply modelled with routine filtering procedures in the undifferenced approach, while the double difference of such a process is hard to describe either in theory or in practice. Furthermore the differencing process entails an obvious loss of information.

Differencing often leads in practice to simplifying assumptions, in order to make it at all possible to estimate relevant parameters. However such simplifications frequently prevent the successful adoption of new models.

4 DGNSS: Observation space or state space?

In contrast with a simultaneous computation of the data from two or more GNSS receivers, in differential (DGNSS) procedures the principle is of corrections to observed quantities. This means that at a reference station the sum of all the error effects operating there is computed and then transmitted to the user as a correction value. The user applies this correction value to his own observation and this improved measured quantity leads in turn to a position fix. If the reference station coordinates are known in terms of the global reference system of the GNSS, the coordinates determined in this way represent an absolute position. The method can also be regarded as a differential procedure in the observation space domain, since a correction is made to the measured quantities.

The principle of operation of DGNSS procedures in the state space domain is different, however. Instead of correcting the observations, the state of the individual error components is determined and made available to the user. If the quantities are estimated with sufficient accuracy, the user can then eliminate the corresponding error term from the observation equation. Most of the terms given in the observation equation can be treated as state parameters in this way. In practice the meaningful physical parameters are those which apply globally or over wide areas. An example of the application of such parameters is the use of the precise ephemeris in post−processing.

The proposal to make use of parameter estimation in the state space domain to support real−time procedures was originally made several years ago (Wübbena et al. 1996a, Wübbena, Bagge 1997, Wübbena 1998), with the presentation of the principle of the GNSS−SMART (State Monitoring And Representation Technique) system for DGNSS and RTK applications. Instead of computing only one correction for the whole signal at a reference station, as in the conventional observation space domain, the state of the system, separated into the individual error components, is determined from the observations of a network of reference stations. This information is then transmitted in a suitable form to the user, so that he can carry out a precise absolute position fix taking account of this data.

Such networks are already in practical use for precise real−time measurements. However, the system state information is currently transmitted in simplified form (as area correction parameters or virtual reference stations) because no international standard for the supply of such data is yet available. The RTCM standard, for example, only allows for the use of observation space, so the system state data must be reduced to observation−related information. Moreover, depending on the extent of the networks, the system state information may only be considered to be valid at a regional level.

Similar approaches are meanwhile also being used by other organisations. Thus the Jet Propulsion Laboratory (JPL) seeks to determine the satellite orbit and clock errors in quasi−real time, and to make use of this state information in the form of corrections to the broadcast ephemeris and clock parameters (Muellerschoen et al. 2001). By these means dual frequency receivers operating globally in kinematic mode can supposedly achieve an absolute accuracy in the region of 20 cm in height and 10 cm in plan position.

The determination of system state information in a GNSS SMART system need not take place in one continuous stream. For example scenarios can be envisaged in which parameters of global character such as satellite orbits and clocks are drawn from global networks, while regional parameters such as wide−area ionospheric delay effects and parameters with limited spatial validity such as local ionospheric disturbances and tropospheric effects are drawn from regional and local networks respectively. One problem with such a method of working is the need for consistency in the data. Because of the sometimes high correlations between the different parameters, this cannot be achieved without additional information. Only through the definition and application of suitable standards, for example for tropospheric refraction corrections, will this kind of solution become useful.

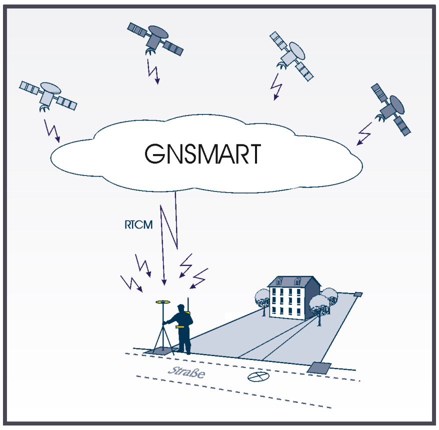

Fig. 1: Absolute positioning in the state space domain

5 Absolute and relative position fixing

An absolute determination of position in the global reference system of the GNSS is only possible to the accuracy with which the sum of all of the error components in the observation equation is known. The random error of measurement (or measurement noise) of a geodetic GNSS receiver for carrier phase observations is of the order of 1 mm or less. A kinematic positioning accuracy of 1 mm is thus possible, if the sum of all external error sources can be determined with an appropriate higher accuracy (amplified by DOP factors).

However this is not yet entirely possible. For GPS code measurement without S/A and with dual−frequency equipment, absolute accuracies reach around 1 to 2 m.

To achieve higher accuracies, up to now RTK procedures have been used, working either with code and phase corrections from a reference station or by carrying out a simultaneous computation using the measured quantities at a reference station and at the mobile station. These both therefore represent relative positioning in observation space.

The introduction of the procedures in the state space domain described earlier (GNSS SMART) will lead more and more in the future to absolute positioning and to the highest accuracy. Instead of relative positioning referred to correction or reference data from a single reference station (in observation space), absolute positioning, referred to system state information from networks of monitoring stations, will be carried out increasingly.

Fig. 1 makes this principle clear. The user observes the signals from individual satellites and receives information on the current state of satellite and atmospheric parameters from a service provider. With this data the user carries out an absolute position determination. No direct connection is made to any reference stations.

6 Separation of error effects

A fundamental problem with the treatment of the various error terms is that a separation of the individual components is difficult because there are sometimes high correlations between the different parameters. This problem is even greater when differencing methods are used.

As an example, consider the problem of the absolute and relative phase centre variations (PCVs) for transmitting and receiving antennas. In the International GPS Service (IGS) it was decided at the beginning of the 1990s that in global networks using a variety of antennas relative calibration values of PCVs should be taken into account. The "Dorne Margolin T" antenna was chosen as the reference antenna. For this type of antenna, apart from the height offsets for the L1 and L2 phase centres, the absolute PCV was treated as zero. Up to the present day all operational computations at IGS have used this assumption. In this way all solutions were consistent, even in comparison with other measurement procedures such as VLBI. Only after the introduction of the new type of satellite in Block IIR were discrepancies detected. Investigations show that because of the high correlation with tropospheric, station height and satellite orbit parameters, neglecting the absolute PCV for the receiving antennas had not up to that time led to inconsistencies in the solutions. For the new satellites, however, only by means of an artificial adjustment of the offset of the antenna from the centre of gravity of the satellite can a consistent solution be achieved; but the value assumed for this offset is unrealistic (Rothacher 2000). The introduction of absolute PCVs for the receiver antennas in the IGS network leads, on the other hand, to inconsistencies with regard to the global scale factor in comparison with VLBI. A solution to this problem presumably lies in the introduction of nadir distance−dependent PCVs for the satellite antennas. The antennas of the Block IIR satellites have been modified compared with the older satellites. A working group has been set up in IGS to investigate the characteristics of the satellite antennas.

This example shows that a separation of the different error components is not successful under certain conditions. An alteration of measurement arrangements or conditions (in this case a new type of satellite) can however offer new possibilities or may necessitate a separation of the effects.

7 Satellite parameters

The determination of precise satellite orbits was already intensively pursued at an early stage of GPS development. In the context of the IGS network, continuous observations and analyses are carried out internationally by various organisations, from which, among other things, precise orbital data is computed. These computations deliver consistent solutions down to the centimetre level.

Because orbit determination is mostly carried out with software systems using "double differencing", for a long time no precise regulation of satellite clocks was carried out. However by using undifferenced code observations precise clock parameters have also been computed over the past few years. Such information is available on an operational basis with only a short delay, for post−processing applications.

The transit time delays in the satellite hardware are not known in an absolute sense. For the delay of the L1 C/A code, before the launch of each satellite a so−called time group delay parameter (T_{GD}) is fixed. This will be transmitted to the user in the satellite message and is intended for the use of single frequency receivers. Corresponding parameters are not supplied for the L1−L2 code differences because such an effect is included in the satellite clock computation procedure of the control segment. However parameters are computed in the IGS for the description of the relative time delays between the C/A code and the P code on L1. This is possible with the help of receivers which can measure using both signals simultaneously.

The missing information on group delays between the codes on L1 and L2 means that an absolute ionosphere cannot be derived directly from the measured quantities; for this an additional ionospheric model is thus necessary.

It is astonishing that up to now practically nothing is known about the effect of phase centre variations at the satellite antenna. Correction models, allowing for example a nadir distance−dependent correction of the phase measurements, are consequently not available. The satellite antenna consists of an array of a total of 12 antenna elements which are arranged in two concentric rings. In this way the requirements for broad−band directional emission characteristics are met. It is known from investigations into receiver antennas that the PCV characteristics can give rise to substantial variations of up to several centimetres. It is therefore to be expected that similar effects also occur at the satellites.

8 Ionosphere

The ionosphere plays an important role in many applications. The use of dual frequency observations allows the first order ionospheric effect on the transit time to be determined and corrected. However the problem arises, especially for observations over short periods and for real−time applications, that before the dual frequency correction can be carried out the carrier−phase ambiguities must be resolved. The ambiguity resolution is not independent of the effect of the ionosphere, which rather must be known or modelled with sufficient accuracy in order to arrive at integer solutions for the ambiguities. Over greater ranges this can sometimes become very difficult, so that wide area and global networks often have to manage without integer ambiguities. One then works with the so−called “ionosphere−free” linear combination L_0. High accuracies are however achieved only after sufficiently long observation periods.

Because of the ambiguity problem, the ionosphere is for many applications the most important limiting factor. This is particularly the case in the present phase of high solar and ionospheric activity.

The third GPS carrier frequency which has already been announced will bring a significant improvement to this situation. Both rapid ambiguity determination over longer ranges and the inclusion of higher order terms will become possible. A complete absolute system state model will also be supported and will become achievable.

9 Troposphere

Tropospheric refraction is a further limiting factor for precise applications, especially for the determination of height. For precise static applications, tropospheric parameters have been modelled virtually since the beginning of GPS development, thereby also achieving high accuracy in the height component. However for kinematic applications such modelling was not usually done, because the system became too weak and the accuracy of heighting was reduced accordingly.

Lowering the elevation mask leads to an improvement in the geometry, but also to increased tropospheric error effects. Better processing using functional and stochastic models of the troposphere and possibly the inclusion of additional meteorological information can produce improved results here.

The troposphere also plays an important role in reference station networks. Of the distance−dependent errors this is the component which shows strongly variable spatial and temporal correlation and which cannot be determined directly but only from the measured quantities over lengthy periods.

Regardless of whether the procedures operate in observation space or in state space, the behaviour of the troposphere must be understood to a sufficient spatial density. However it is advantageous to work in the state space domain because the information does not merely relate to single satellites but can be applied to other satellites, a solution which is not possible in observation space.

Apart from the need to determine the tropospheric error component in order to be able to carry out precise position fixing, the relevant parameters also provide valuable information about the meteorological conditions in the lower atmosphere. The results are therefore of interest to weather forecasting services, especially when they are available in real time.

10 Multipath effects

Multipath effects represent the most important limiting factor for accuracy in many applications. The expression “multipath” (MP) is used to refer both to the effect itself and to the resulting error.

The error caused by multipath effects is dependent on the surroundings of the receiving antenna (that is, the geometry and nature of reflective objects) and on the antenna itself, as well as on the signal processing in the antenna and in the receiver. As well as reflections, diffraction phenomena are counted among multipath effects. Reflections of the transmitted signal at the satellite should also be considered.

For carrier phases the MP error has a strongly local character. This is because it results from the superimposition of the direct and the reflected signals. Because of the wavelength of about 20 cm, small movements of the receiver antenna can cause large changes in the relative phase of these signals, so that the error effect also suffers large variations.

The temporal variations of MP effects for static measurements are however often small, because the geometry of the indirect signals varies only with the movement of the satellite. The angular velocity of the satellite at a receiver antenna on the surface of the earth is very small, so the geometry of the indirect signal changes only slowly. If the indirect path changes through one wavelength, a complete cycle of error is received provided that the reflection characteristics do not change. Typical periodicities are of the order of a few minutes. The period diminishes in proportion to the distance from the reflecting surface.

Different approaches are possible for the mitigation of multipath problems and are used by different organisations. These include:

- signal processing techniques,
- the exploitation of spatial variations,
- the exploitation of temporal variations,
- the exploitation and combination of measured quantities.

While different signal processing techniques enable a reduction of MP in code measurements, up to now they have provided no solution to the problem for carrier phases.

Over a small area the spatial variations can be detected either through using several antennas (or antenna arrays), or by moving one antenna. Under the assumption that all other error sources at the location continue to act in the same way and that the relative positions of the measurements are known, the differences in the multipath effect can be measured. If measurements are taken at a sufficient number of positions, under certain model assumptions the absolute effect of multipath can in theory be determined (Ray 2000).

If measured simultaneously at several antenna positions in close proximity, it cannot be assumed that the MP is the same as for a single antenna, but a calibration is possible for a corresponding measurement arrangement. If measurements are taken at successive epochs with a moving antenna, a precise measurement of the relative positions by other methods is required. The movement must be able to be carried out quickly in order to minimise the effect of temporal variations in MP. In any case such measurement arrangements involve greatly increased expenditure including the cost of special hardware. Thus at present their practical economic application is limited to special cases such as the calibration of permanent reference stations. However valuable knowledge of multipath behaviour can be gained in this way.

The movement of a receiving antenna through a distance of one wavelength of the signal leads, as already explained, to one complete oscillation of the MP error. This fact can be exploited, for example to speed up and to stabilise the initialisation of an RTK solution in surroundings with strong multipath effects, where a fixed installation can lead to longer initialisation times and incorrect ambiguity determination because of systematic errors. By moving the antenna the multipath effect takes on a random character. The disadvantage of the smaller redundancy of kinematic observations is often less important in such cases.

The temporal variations of multipath are smaller. A complete period covers at least several minutes for individual satellites. For different satellites the periods will vary so that this error component can be successfully reduced by averaging over longer observation times.

The slow variation, that is the high temporal correlation, of multipath effects can however be exploited for the precise determination of small movements, over short periods (of the order of seconds) or even at high epoch rates. On the other hand slow movements are not exempt from multipath effects which are thus harder to detect.

Multipath effects are repeatable provided that the surroundings of a receiving antenna do not change. This means that two satellites observed at the same azimuth and elevation are subject to the same error effect. The configuration of the GPS satellite constellation is repeated at intervals of one sidereal day, so multipath errors are also repeated on successive days. This fact can be exploited, for example small changes in position from day to day can be determined very accurately. Various investigations have shown that sub−millimetre accuracies can be achieved in this way.

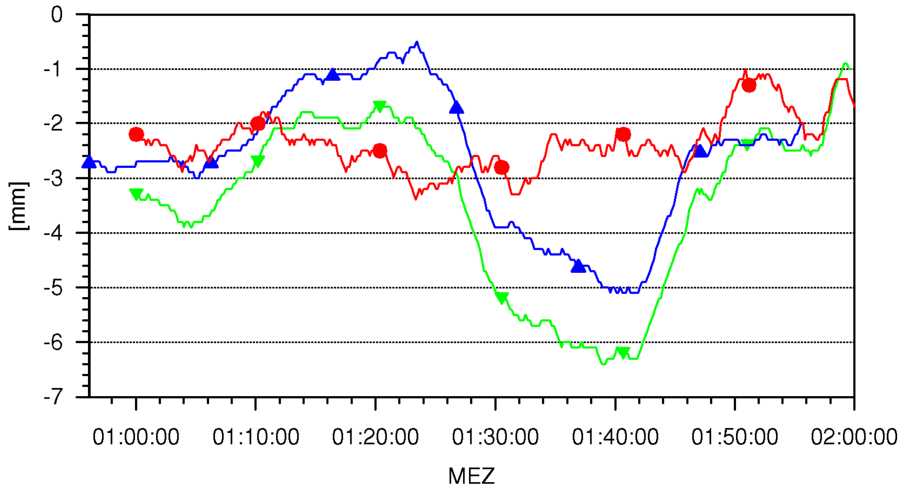

Fig. 2: Multipath reduction with sidereal differences in observation space, height (sidereal) circular symbol, height (first day and second day)triangular symbol for Central European Time

Fig. 2 shows an example of a kinematic analysis of a static measurement carried out on two successive days. The height component of the RTK solution is shown, filtered with a small time constant. The graphs with triangular symbols show the solutions for the two days. These show variations of the order of about 5 mm which can be attributed to multipath effects. The curve shown with circular symbols indicates the height variation between the two days deduced from the sidereal phase differences. The change of about 2 mm in the height of the point between the two days can thus be derived with an accuracy of better than ± 1 mm.

Sidereal day differences and the associated elimination of multipath errors were also exploited in the development of the procedure for the absolute field calibration of GPS antennas (Wübbena et al. 1996b).

A calibration of static antenna installations also seems possible using this assumption. However a problem arises from the dynamic alteration of the surroundings of a receiver antenna through varying meteorological conditions and the resulting changes in the reflection properties.

The development of models for the description of multipath effects is made more difficult because a separation of the error components from other quantities is only possible with costly measurement arrangements. The frequently used simple analysis of static double differences allows an assignment of observed errors to individual stations and satellites only under certain assumptions. It is often assumed that satellites at high elevations are free of multipath errors (Wanninger 2000). Assumptions of this kind are frequently found not to apply at reference stations with surfaces such as pitched roofs in the neighbourhood. Because of the reflection geometry, in this case high satellites are often more severely affected than are low ones. With double differences there are always two stations involved, so an explicit assignment to one station is also not possible. Here the assumption is frequently made that particular stations or particular station means are free of MP. The transfer of the information to other pairs of stations here seems questionable.

The error caused by MP is dependent on the signal used. This fact can be exploited in order to derive the error component directly from the measured quantities (Ray 2000). However, combining code and carrier phase measurements leads to the problem that code measurements in particular are significantly modified by unknown algorithms in so−called “tracking loops” in the receivers. The error effect of multipath is therefore no longer included in its original form. The combination of the L1 and L2 carrier phase measurements also only offers limited possibilities of reaching a conclusion on multipath effects because separating them from ionospheric effects is so difficult.

The signal−to−noise (S/N) ratio of a GNSS receiver provides further measurement information. Theoretical and practical considerations indicate that the S/N values vary due to multipath effects. In comparison with measurements without multipath effects the following behaviour is revealed. With maximum multipath error the S/N ratio is no different from the value without multipath effects. The maximum change in S/N ratio occurs with minimum multipath error. The latter situation arises when, for example, the indirect signal has the same phase as the direct signal. The signals are additive in this situation so that the S/N ratio is correspondingly increased. However there is no error in the resulting phase measurement. If the phases are shifted 180° apart the S/N ratio is reduced but here also the phase measurement remains error−free. Approaches which weight the observations in accordance with their S/N ratios in order to minimise the effects of multipath on the measurements seem rather doubtful in the light of this explanation. Such an approach theoretically succeeds only for long−period static measurements where, for example, a uniform weight is applied to all observations of one satellite on the basis of temporal variations in the S/N ratio due to multipath effects. In kinematic applications and with rapid static measurements such approaches fail.

The signal−to−noise ratio is a quantity which is not standardised. Depending on the manufacturer or the receiver model, clear differences can be found in the measurement of the S/N value. This makes the use of this observable even more difficult.

11 Antennas

The problem of phase centre variations (PCVs) in receiving antennas has essentially been solved for work with high accuracy requirements (PCVs < 1 mm) (Wübbena et al. 2000b). It will be discussed in detail in this volume by Schmitz (2001).

12 Receiver parameters

These refer essentially to the transit time delays in the hardware and the receiver clock. So−called "inter−channel biases", as produced earlier in some types of equipment, are no longer of practical relevance today on account of the extensive use of digital signal processing. Where the signal processing is still shared among different hardware components the equipment itself carries out a calibration so that further processing can be undertaken without needing to consider the terms involved.

When different signals are considered it is however necessary in general to take account of the corresponding bias. This applies especially to the processing of signals on different frequencies. For this reason an absolute determination of, for example, the ionospheric delay is only possible to a very limited extent with code measurements.

The oscillators for the generation of the reference signals and of receiver time generally possess good short−term stability, but are not stable in the longer term. The corresponding clock model therefore leads only to a small gain of information. In practice it follows that for precise applications a new clock unknown should be determined for each measurement epoch. A different situation arises if the receiver oscillator is supported by an external atomic clock frequency.

13 Summary

Even after 20 years of GPS and more than 15 years of intensive geodetic application there is still a wide range of questions in need of answers, and problems requiring solutions, in order to make further improvements in the application of GNSS procedures to precise positioning. For real−time applications a transition is becoming apparent, from procedures operating in "observation space" to those which work in the "state space" domain.

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